

# Model Checking

Boris Feigin

`b.feigin@cs.ucl.ac.uk`

University College London

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# Introduction

## In a nutshell...

*Model checking* is a collection of techniques for *automated formal verification* of finite-state concurrent systems.

Best suited for analysis of...

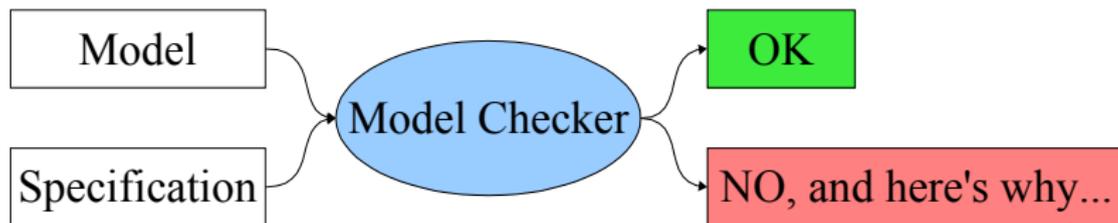
## *Reactive systems*

Characterized by continuous interaction with environment.  
Control-oriented.

- Hardware controllers
- Protocols of various kinds



# Basic Operation



- *Model* — usually, an abstraction of the system being analysed (for example, using some process algebra or even UML)
- *Specification* — properties the system must satisfy (e.g. absence of deadlocks, liveness, invariants etc.), expressed in some suitable formalism

# Transition Systems (Model)

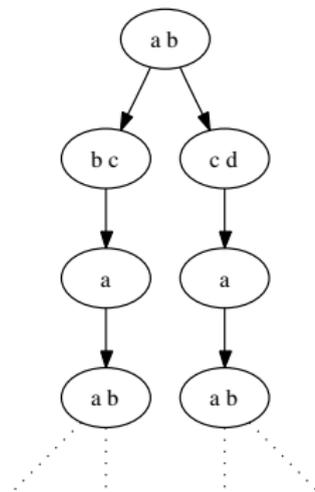
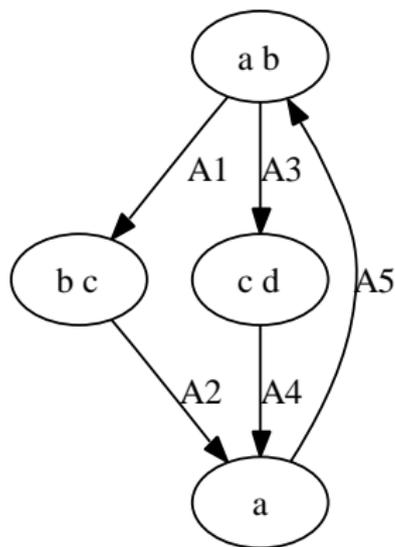
We reason about reactive systems in terms of their *state* and hence model their behaviour using *state transition systems*.

## Definition [Müller-Olm et al.]

A *Kripke transition system*  $T$  over a set of *atomic propositions* AP is a four-tuple  $(S, \text{Act}, \rightarrow, I)$  where

- $S$  — set of states
- $\text{Act}$  — set of actions (e.g. program statements)
- $\rightarrow \subseteq S \times \text{Act} \times S$  — transition relation
- $I : S \rightarrow 2^{\text{AP}}$  — *interpretation*;  $I(s)$  for some  $s \in S$  is the set of propositions which are true in  $s$  (e.g.  $a = 1$ )

## Illustration



We can *root*  $T$  with an initial state  $s_0 \in S$  and unfold it into an infinite *execution tree* (you'll see later why we might want to do this)

# Example

We'll use the *finite state processes* (FSP) process algebra from concurrency theory to describe a *Labeled Transition System* for the Dining Philosophers Problem.

*Demo using LTSA example for Dining Philosophers*



# Temporal Logics (Spec)

- Natural formalism for expressing assertions about system evolution
- Differentiate between
  - *Linear-time* — consider *linear* paths in execution tree
  - *Branching-time* — quantify over paths
- Construct formulae from *atomic propositions* and *boolean and temporal connectives*



# Propositional Linear-Time Logic (PLTL)

- Important operators:
  - **X**  $\varphi$  (“next  $\varphi$ ”)
  - $\varphi$  **U**  $\psi$  (“ $\varphi$  until  $\psi$ ”)
  - **F**  $\varphi$  (“eventually  $\varphi$ ”) — liveness
  - **G**  $\varphi$  (“always  $\varphi$ ”) — safety
- Used to express *correctness* properties of the system



# Illustration

NEXT(P)



UNTIL(P, Q)



EVENTUALLY(P)



ALWAYS(P)

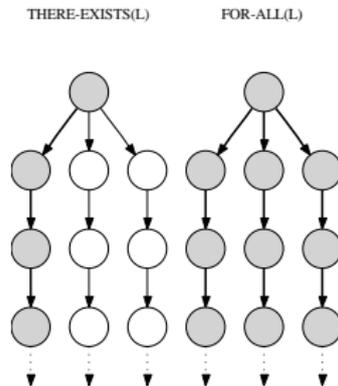


# Computation Tree Logic (CTL)

- Introduce *selectivity*: “there exists” **E** and “for all” **A**
- Path quantifiers above are combined with PLTL operators



# Illustration and Examples



## Examples

(from Merz) **AG**  $\neg(\text{owns}_1 \wedge \text{owns}_2)$  — mutual exclusion

(from Clarke et al.) **AG (EF Restart)** — can always restart

# Model Checking

Given a transition system  $T$  and a temporal logic formula  $\varphi$ , the model checker decides whether  $T \models \varphi$  is true (“ $T$  is a model of  $\varphi$ ”)

Algorithms fall into two categories:

- Local — used with **PLTL**
- Global — **CTL**



# The State Explosion Problem

## Major Obstacle

Complex systems have astronomical numbers of states

The problem of *state explosion* plagues approaches relying on explicit construction of states

Also worth noting that,

- In practice, descriptions in high-level modeling languages are used in preference to low-level constructs such as those discussed
- HOWEVER, the size of transition systems derived from such descriptions grows *exponentially* with the length of the description



# Techniques

- *Partial Order Reduction*
  - Consider concurrent processes —  $T$  contains all possible interleavings of actions of individual processes
  - Provided processes are loosely coupled, may be able to exploit commutativity of actions to remove a lot of redundancy
- *Abstraction*
  - Omit superfluous detail which does not affect the property being checked
- *Symmetry Reduction*
  - Exploit structural regularities in the system



# Symbolic Model Checking

## Thinking out of the box

Can explicit construction of states be avoided altogether?

- *Binary Decision Diagrams (BDDs)* — an efficient encoding of boolean formulae — serve as implicit representation of states and transitions
- Temporal formulae can be model-checked on BDDs directly
- Hence, can handle much larger numbers of states



## Example Applications

- Fluke microkernel IPC subsystem
- BB84 quantum key distribution scheme — see issue 11.3 of *ACM Crossroads*
- “Remote Agent” spacecraft controller



# Software

- SPIN — accepts PROMELA as the input language
- Java PathFinder — works on level of Java bytecode; authors report successes with up to 100KLOCs
- Bandera tool set
- LTSA — uses FPS process algebra



# Model Checking Vs. Deductive Verification

*Deductive verification (or theorem proving)* takes a different route: correctness proofs are constructed from axioms by application of inference rules.

## Model Checking

- Automated — almost a “black box” tool
- Limited by state explosion
- “Brute force”

## Deductive Verification

- Manual — requires substantial knowledge and experience
- Can handle infinite-state systems
- “Intelligent”



# Summary

## In a nutshell...

*Model checking* is a collection of techniques for *automated formal verification* of finite-state concurrent systems.

## Pros

- Automagic
- Gives counterexample on violation of spec.
- Widely used: controllers, protocols, operating systems...

## Cons

- State explosion limits applicability
- Beware human errors: poor modeling, incomplete spec. etc



## Further Reading

-  E. M. Clarke, O. Grumberg, D. A. Peled  
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