Evolving gzip matches Kernel from an nVidia CUDA Template

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Abstract—Rather than attempting to evolve a complete program from scratch we demonstrate genetic interface programming by automatically generating a parallel CUDA kernel with identical functionality to existing highly optimised ancient sequential C code. Generic GPGPU nVidia kernel C++ code is converted into a BNF grammar. Strongly typed genetic programming uses the BNF to generate compilable and executable graphics card kernels. Their fitness is given by running the population on a GPU with randomised subsets of training data itself given by running the original code's test suite.

I. Introduction

The goal of genetic interface programming applied to Software Engineering (GIPSE) is not to evolve complete systems but to automatically generate small interfaces between or components of much larger systems which have already been created by traditional software engineering techniques, principally human coding. There are many areas of software where quite small software units are required to fit a well define interface. In many cases it will be simplest to write the code. However as systems get more complex finding a person with skill's appropriate to the two or more potentially very large subsystems being interfaced becomes more difficult. This is especially true where the subsystems are written in different languages or by different suppliers or reside on different hosts.

The requirement becomes still more difficult for the programmer when the requirements change or involve multiple competing interests. In a traditional server host there may be little incentive to produce compact code, whereas in a single user mobile device space and battery life might need to be juggled against less severe response time. In principle a multi-objective automated system could come up with a range of Pareto optimal implementations allowing the software engineer to choose the solution appropriate to the current requirement. Indeed when requirements change the software maintenance engineer might choose a different solution from the existing non-dominated set, or even re-run the evolution with new requirements.

As a very first demonstration of these ideas we have taken a well constrained problem. Evolving an nVidia CUDA graphics card kernel to parallelise existing sequential code. Creating high quality GPGPU kernels is known to be difficult and there are, as yet, few experts in CUDA. In software engineering this is not uncommon. Indeed it has been known for systems to be constructed, not in the computer language of choice, but the language where there is a ready supply of skilled programmers.

The Software-artifact Infrastructure Repository (SIR) [Hutchins et al., 1994] contains a version of gzip suitable for large scale regression testing and a test suite for it. A large part of the computational load during file compression, is scanning the input looking for identical strings. (gzip compresses files by replacing long string which occur multiple times by much shorter codes.) In principle, for every byte in the input file, gzip needs to find all the parts of the file which match it. It then remembers the longest match, which may then become one of the compression codes. gzip incorporates hashing and numerous heuristics and restricts the search window to limit the computational load. These heuristics have been tuned by hand. A possible future investigation would be to consider re-tuning gzip in the light of modern demands. However we have not tried to do this, instead we seek a CUDA C++ kernel which is not only compatible with existing versions of gzip and compressed files but insist the new graphics code yield identical outputs.

The next section describes how we start with code supplied by the manufacture nVidia for a totally different application and use it as a template for a gzip string matching CUDA kernel. Section III-A describes its conversion into a context free BNF grammar. Section III-B says how the existing SIR test suite for gzip was converted into a GP fitness function [Poli *et al.*, 2008]. The evolution and validation of the automatically created CUDA gzip kernel are described in Sections IV and V. We finish with a discussion of future work (Section VI) and our conclusions (Section VII).

II. ANALYSING HUMAN EXAMPLES

We start, as a human novice might, by studying the available example codes. CUDA 2.3 comes with 67 examples code in C++. scan_naive_kernel.cu was selected. The comments were stripped from it and it is was converted to take the right inputs and operate on the right types for a gzip matches kernel. The human created starting point for the automatic generation of a CUDA kernel to replace the gzip longest_match routine is shown in Figure 1.

As a first step towards the evolution of CUDA code, the template kernel was deliberately kept simple. By removing direct access to __shared__ data and threading information, thousands of GP individuals can be individually tested in parallel on the GPU without the risk of them interfering with each other. This does, however, restrict the type of kernels which can be evolved. Again it must be stressed that at this stage we seek only to show the approach is possible. As we

```
_device__ void scan_naive(int *g_odata, const uch *g_idata, const int strstart1,
const int strstart2)
    //extern shared float temp[];
   int thid = 0; //threadIdx.x;
   int pout = 0;
   int pin = 1;
   int offset = 0;
   int num_elements = 258;
    <3var> /*temp[pout*num_elements+thid]*/ = (thid > 0) ? q_idata[thid-1] : 0;
    for (offset = 1; offset < num_elements; offset *= 2)</pre>
        pout = 1 - pout;
       pin = 1 - pout;
        //temp[pout*num_elements+thid] = temp[pin*num_elements+thid];
        <3var> = g_idata[strstart+pin*num_elements+thid];
        if (thid >= offset)
        <3var> += g_idata[strstart+pin*num_elements+thid - offset];
   return <3var>;
}
```

Fig. 1. Template. Based on scan_naive_kernel.cu supplied with CUDA 2.3 by nVidia. Simplified by removing direct access to thread index threadIdx.x, __shared__ data temp (and hence __syncthreads()) and replacing function inputs and direct write to g_odata[threadIdx.x] with return. Convert from float to int and uch data types. The three places where temp was written are replaced by writing to an int local variable (<3var>). Where elements of temp were read, temp is replaced by the only other array g_idata.n is renamed num_elements. To avoid complication between statements inside for loop and outside the loop control variable offset is declared at the start of the function rather than in the for statement.

said in the introduction, we do not yet try to demonstrate automatically generating efficient, high performance code or improved compression by finding more or better matches or other trade-offs between multiple objectives.

The CUDA 2.3 routine contained three lines where temp[pout*num_elements+thid] was written to but to simplify the new template __shared__ array temp has been removed. The new template was written to ensure that the three places where element pout*num_elements+thid of temp was modified are each replaced by modifying the same variable. In principle any of the modifiable scalar variables in scope could have been used. (This is denoted by the rule <3var>.) However in a context free grammar, it is tricky to ensure that once a grammar rule has been used it always expands to the same choice. Hence, although <3var> should be any variable, it simply expands to the first. (Although <3var> is present in the evolved solutions, Figures 20 and 22, it is not useful.)

There are too many occurrences of the letter "n" in the grammar, therefore the variable n was renamed num_inputs. Its default value, 258, is taken from gzip. As is usual with GP [Langdon, 1998], values, including constants, can evolve. Thus the grammar provides the means for introducing a small number of int and uch constants which can be used directly or as part of evolved expressions. 0, 1 and 2 come from the original CUDA code, whereas 258 comes from gzip.

III. PREPARING GENETIC PROGRAMMING

In addition to the standard five steps described by Koza [Koza, 1992], the strongly typed [Montana, 1995] grammar based GP [Langdon and Harrison, 2009] needs a BNF grammar. The next section describes the motivation behind and the mechanism for, the conversion of the human template (Figure 1) into a context free grammar (Figures 2 to 8).

A. The Grammar

The automatic generation of CUDA Kernel's was in part motivated by last year's prize winning work by Westley Weimer [Weimer et al., 2009] in which bug fixes were automatically evolved by genetic programming. Whereas in [Weimer et al., 2009] the GP works by automatically reusing lines of human written code from elsewhere in the faulty code, we use a BNF grammar to describe a much smaller unit of human code. (I.e. the nVidia code used as the template.) The grammar also allows the GP much finer control. Instead of reusing whole statements, the grammar allows fragments of lines and even individual expressions to be be manipulated. At the same time the combination of grammar and strong typing ensures (unlike [Weimer et al., 2009]) that each of the genetically manipulated individuals is legal CUDA C++ code. They compile and run and produce answers. There is of course, as usual, no guarantee that they do anything sensible.

We use gawk in conjunction with Unix shell scripts. Firstly, to help create the grammar and secondly, to do all the genetic manipulations (crossover and mutation) and C++ code generation [Langdon and Harrison, 2008].

As with our previous work on using GP to evolve variants of complete human written C code for mutation testing [Langdon et al., 2009], we start by converting the human written code line-by-line into a BNF grammar (Figure 2). Next comes the grammar start rule (<start>) and the hierarchical sequence of rules which link it to the individual lines (i.e. the complete CUDA kernel, Figure 3). In the work on inserting mutations into existing code [Langdon et al., 2009], only small changes were permitted and the structure of the code, including the order of the lines of code was fixed to to be identical of the original human written code. However since we seek to evolve new code, this grammar allows the lines to be omitted, to be reversed and structures, like for and if can be nested.

To ensure the new code compiles some of the structure of the template (Figure 2) is enforced by the grammar. Although the C++ syntax allows fairly free ordering of variable declarations, variables must still be declared. To allow this constraint to be simply enforced by the grammar, the grammar is written so that all the variables are declared at the start of the CUDA kernel. Similarly the grammar ensures that it ends with a return statement.

As with [Langdon and Harrison, 2008] the right hand side of each BNF is restricted to binary alternative rules (I.e. one | separator per line.) Of course a rule can also be a single list of terminals and rules to be expanded, without alternatives. Two alternatives per rule allows each GP individual to be represented internally as a binary tree. However this is not a fundamental restrictions in two senses. 1) A higher arity tree can always be represented by a (larger) binary tree. 2) The implementation could readily support more alternatives per BNF production. For implementation reasons, the empty string "" is used to represent the null alternative \$.

We adopt the convention that the alternative closer to the original code is usually the first alternative. The first alternative often consists mostly of terminals and so represent the smaller grammar sub-tree. This is useful when the genetic operations start to run into space or depth limits. If such limits are reached, the genetic operations can chose the smaller option in an expansion without the need to investigate all options and back track to the smallest one. The fragment of the grammar given in Figure 3 shows that line6e> can either expand to give code identical to line 6 of the template or something similar. In the second case the declaration of int pout includes initialising pout with an evolvable constant. (Note the use of <intconst> ensures the constant is of the same type the variable pout.)

Line 8 if the template is fairly complicated. The first three lines of Figure 5 say it may be omitted (i.e. replaced by "") used unchanged or modified. Rule line8.0.0> breaks line 8 into its four components.

Again each of the components has alternatives which are the same as the template code and others which progressively generalise it. E.g. line8.2.1.2.2> gives the most general form of the original code thid > 0 as <intvar> <compare> <intconst>.

Line 10 contains a for loop. Figure 6 contains other examples of how the template biases the grammar but the grammar also allows generalisations of it. For example, the loop continuation test in line 10 uses <, rule 10.2 says < may be used but also allows any comparison. As another example, the template uses *=2 in the iteration step but rule 10.4 allows ++ etc.

Line 16 is a conditional statement. Again the grammar allows it to be included, omitted, modified and moved (Figure 7). The final part of the grammar (Figure 8) contains most of the generalisation. Note the grammar still tends to favour expressions of the same type and rules like <intspecialexpr> encourage the reuse of expressions created by the authors of CUDA.

B. The Fitness Function

1) The Training Data: SIR supplies a test suite of 214 tests for gzip. We used 211 of version 1.4 of these. (The remaining three had proved to be unsuitable for using in automated scripts, e.g. due to non-determinism introduced by behaviour which depends on the detailed timing of events in the test script.) We extended our gzip test bed (used in mutation testing [Langdon et al., 2010b]) to instrument all the inputs and outputs of gzip's longest_match routine. When running our SIR test suite longest_match is called 1599 028 times and makes 42 472 851 searches.

gzip uses a sliding window of 64kb in which it searches for repeats of each string in the file to be compressed. It greatly reduces the 2147450880 potential matches principally by hashing. Nevertheless for each byte in the file longest_match still has to search every item on a particular hash chain. For pragmatic reasons gzip limits the length of matches to between three and 258 bytes.

Figure 9 shows that the number of number of items on the hash chain when running the 211 SIR tests. The number times longest_match is given a hash chain of a given length falls very rapidly with its length. So although the maximum is 1588, the median is only eight and in $\approx 200\,000$ calls there is only one item to be checked. To reduce the volume of test data and give a more uniform spread of training data, for each of the 211 SIR tests only the first of each call of longest_match with a particular number of parallel searches was kept and thus available to train the GP. This yielded 29 315 examples (dashed line in Figure 9).

Each generation 100 of the 29 315 examples were randomly chosen (without reselection). Each generation the fitness of every evolved CUDA kernel in the population was assessed on the same one hundred examples. The distribution of number of parallel searches in a given generation follows the lower line in line in Figure 9 (together with random sampling noise). Thus the number of times each GP individual is called will be the same in a given generation but

```
1 < 2 >
          ::= "_device_ int kernelXXX(const uch *q_idata, const int strstart1,
             const int strstart2) \n"
          ::= "{\n"
1 < 3 >
          ::= "int thid = 0;\n"
e6>
          ::= "int pout = 0;\n"
::= "int pin = 1;\n"
line71>
          ::= "int offset = 0;\n"
1
          ::= "int num_elements = 258; \n"
e8>
          ::= <3var> "= (thid > 0) ? G_idata(" <strstart> "thid-1) : 0; \n"
line10>
          ::= "for (offset = 1; offset < num_elements; offset \star= 2) \n"
line11>
          ::= "{\n" "if(!ok()) break;\n"
          ::= "pout = 1 - pout; \n"
12>
line13>
         ::= "pin = 1 - pout; \n"
<line15> ::= <3var> "= G_idata(" <strstart> "pin*num_elements+thid);\n"
<line16> ::= "if (thid >= offset) \n"
line17>
          ::= <3var> "+= G_idata(" <strstart> "pin*num_elements+thid - offset);\n"
line18>
          ::= "}\n"
line20>
          ::= "return" <3var> ";\n"
          ::= "}\n"
<3var>
          ::= "thid"
<strstart> ::= "strstart1+" | "strstart2+"
```

Fig. 2. First part of grammar used to evolve CUDA matches kernel for gzip. These rules allow little variation but reproduce the initial template. XXX is replaced by the GP individual's identification number. ok () is a macro which prevents infinite loops. <3var> could be any of the three variables pout, num_elements or thid but only thid was implemented.

```
<start>
                       ::= <line2> <line3> <line5> <line6e> <line7e> <line71e> <line72e>
                              <line8e> <line10-20> <line21>
10-20>
                       ::= <line10-18> <line20e>
                                                "" | <line10-18a>
10-18> ::=
<line10-18a> ::= <line10e> <line11> <forbody> <line18>
                      ::= <line12-17a> | <line12-17b>
12-17>
<line12-17a> ::= <line12-13> <line15-17>
<line12-17b> ::= <line15-17> <line12-13>
<line12-13> ::= <line12-13a> | <line12-13b>
<line12-13a> ::= <line12e> <line13e>
<line12-13b> ::= <line13e> <line12e>
<line15-17> ::= <line15-17a> | <line15-17b>
<line15-17a> ::= <line15e> <line16-17>
<line15-17b> ::= <line16-17> <line15e>
<line16-17> ::= <line16-17a> | <line17.0>
<line16-17a> ::= <line16-17.1> | <line16-17.2>
<line16-17.1> ::= <line16e> <line17.0>
17.2> ::= 16-17.2> ::= 16-17.2> ::= 17.2> ::= 18.2
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                       ::= <line12-17> | <forbody.1>
<forbody>
<forbody.1> ::= <forbody.1a> | <forbody.1b>
<forbody.1a> ::= <line8e> <line12-17>
<forbody.1b> ::= <line12-17> <line8e>
```

Fig. 3. Second part of grammar. <start> is the first rule of the grammar. These rules allow lines of code (defined in Figure 2) to be used, excluded, reversed and to be nested. Note using the first of each binary option tends to give a GP individual closer to the template. The function header, int declarations (<line2> ... line72e>) and the return statement (<line20>) are always included.

Fig. 4. Mid portion of grammar. The first 8 rules allow variation in the initial value assigned to each variable. statement.

```
"" | <line8.0>
e8e>
                                                          ::=
< line 8.0 >
                                                          ••=
                                                                                                                                    10.0>
                                                           ::= <3var> "=" "(" <line8.2> ")" "?" <line8.3> ":" <line8.4> ";\n"
0.0>
1.1.1.1>
                                                           ::= "pout*num_elements + thid" | !:= "line8.1.1.1.1.2>
                                                                   <intexpr> <plusminus> "thid"
1.1.1.1.2> ::=
                                                                                                                          "thid > 0" | e8.2.1>
1ine8.2>
<1ine8.2.1>
                                                                                                               <line8.2.1.1> | <line8.2.1.2>
                                                           ::=
                                                                       "thid" <compare> "0"
1.1>
                                                           ::=
1.2>
                                                                                                        <line8.2.1.2.1> | <line8.2.1.2.2>
                                                          ::=
1.2.1.
                                                                      "thid" <compare> <intconst>
                                                          ::=
1.2.2>
                                                          ::=
                                                                      <intvar> <compare> <intconst>
                                                                                                                      1 < line 8.3.0 > | < line 8.3.2 
<1ine8.3>
                                                           ::=
<1ine8.3.0>
                                                                      "G_idata(" <strstart> "thid-1)"
                                                           ::=
<1ine8.3.2>
                                                           ::= "G_idata(" <strstart> <line8.3.2.1> ")"
                                                                                                                         <thidexpr> | <intexpr>
1.2.1>
                                                           ::=
                                                                                                                                        "thid" | <thidexpr2>
<thidexpr>
                                                           ::=
<thidexpr2>
                                                                      "thid" <plusminus> <intexpr>
                                                           ::=
                                                                                                                                                  "0" <line8.4.1>
11ne8.4>
                                                           ::=
                                                                                                                         <uchconst> <line8.4.1.1>
1 ine8.4.1>
                                                           ::=
1.1>
                                                           ::=
                                                                                                                         <uchexprT> | <uchexpr>
```

Fig. 5. Grammar for variants of line 8. Again the left hand option tends to be closer to the CUDA template. Although C++ allows fairly free mixing of int and uch (unsigned char) the grammar separates them.

```
line10e>
            ::=
            ::= "for (offset =" <line10.1> ";" <line10e.2> ";offset" <line10.4> ")\n"
line10e1>
10.1>
                 <line10.1.1> | <intexpr>
            ::=
                          "1" | <intconst>
1.1> ::=
10e.2>
            ::= <line10e.2.1> | <forcompexpr>
                "offset" <line10.2> <line10.3>
10e.2.1> ::=
10.2>
                          "<" | <compare>
            ::=
10.3>
            ::=
                 <line10.3.1> | <intexpr>
<line10.3.1> ::= "num elements" | <intconst>
10.4>
                       "*= 2" | <intmod>
            ::=
                         "++" | <intmod2>
<intmod>
            ::=
<intmod2>
            ::= "*=" <intconst>
```

Fig. 6. For loop grammar

```
12.2> 
line12e>
                                                                      ::=
                                                                                                                                                                                   "" | <line12.2.1>
12.2>
                                                                      ::=
12.2.1>
                                                                                 "pout =" <intexpr> "; \n"
                                                                      ::=
                                                                                                                                                                line13e>
                                                                      ::=
                                                                                                                                                                                   "" | <line13.2.1>
13.2>
                                                                      ::=
13.2.1>
                                                                                 "pin =" <intexpr> "; \n"
                                                                      ::=
e15e>
                                                                                                                                                                                   "" | <line15.0>
                                                                      ::=
100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 100 < 10
                                                                                 <3var> "=" <line15.2> "; \n"
                                                                      ::=
1.1.1.1.1>
                                                                                                       "pout*num_elements + thid" | elements + thid | <le>elements + 
                                                                      ::=
1.1.1.2>
                                                                                <intexpr> <plusminus> "thid"
                                                                      ::=
15.2>
                                                                                                                                                   <line15.2.A> | <line15.2.1>
                                                                      ::=
11ne15.2.A>
                                                                                 "G_idata(" <strstart> "pin*num_elements + thid)"
                                                                      ::=
15.2.1>
                                                                                                                                                   <line15.2.0> | <uchexprT>
                                                                      ::=
"G_idata(" <strstart> <line15.2.1.1> ")"
                                                                      ::=
1.1>
                                                                                                                                       <line15.2.1.1.1> | <intexpr>
                                                                      ::=
                                                                                                          "pin*num_elements + thid" | line15.2.1.1.2>
1.1.1.1>
                                                                      ::=
1.1.2>
                                                                                <intexpr> <plusminus> "thid"
                                                                      ::=
                                                                                                                                                                                   "" | <line16.0>
1ine16e>
                                                                      ::=
10.0>
                                                                      ::=
                                                                                 "if (" <line16.0.0> ") \n"
10.0.0>
                                                                                                                                             <line16.0.0.0> | <compexpr>
                                                                      ::=
10.0.0.0>
                                                                                 <line16.1> <line16.2> <line16.3>
                                                                      ::=
                                                                                                                                                                       "thid" | <intvar>
10.1>
                                                                      ::=
                                                                                                                                                                             ">=" | <compare>
11ne16.2>
                                                                      ::=
                                                                                                                                                                "offset" | <intexpr>
10.3>
                                                                      ::=
                                                                                <3var> <modassign> <line17.2> ";\n"
::=
                                                                                                                                 <line17.1.1.1.1.1> | <line17.1.1.1.1.2>
::=
"pout*num_elements + thid"
                                                                      ::=
1.1.1.1.2>
                                                                      ::=
                                                                                <intexpr> <plusminus> "thid"
                                                                                                                                                   <line17.2.A> | <line17.2.1>
110.2>
                                                                      ::=
1.2.A>
                                                                                 "G_idata(" <strstart> "pin*num_elements+thid - offset)"
                                                                      ::=
1.1>
                                                                                                                                                   <line17.2.0> | <uchexpr>
                                                                      ::=
"G_idata(" <strstart> <line17.2.1.1> ")"
                                                                      ::=
1.1.1>
                                                                                                                                       <line17.2.1.1.1> | <intexpr>
                                                                      ::=
                                                                      ::= "pin*num_elements + thid - offset" | <line17.2.1.1.1.2>
1.1.1.1>
                                                                                                                          <line17.2.1.1.1.2.1> | <line17.2.1.1.1.2.2>
1.1.1.2>
                                                                      ::=
1.1.1.2.1>
                                                                      ::=
                                                                                 <intexpr> <plusminus> "thid" <plusminus> "offset"
1.1.1.2.2>
                                                                                                                    <line17.2.1.1.1.2.2.1> | <line17.2.1.1.1.2.2.2>
                                                                      ::=
<line17.2.1.1.1.2.2.1> ::=
                                                                               <intexpr> <plusminus> "thid"
<line17.2.1.1.1.2.2.2> ::= <intexpr> <plusminus> "offset"
```

Fig. 7. Grammar for lines 12-17. I.e. lines taken from inside the template's for loop.

vary randomly between generation. Each time it is called it is given the address of the current string and that of a putative match. (Actually a point in the file with the same hash value.) The individual should return the number of bytes for which the two locations match. The distribution of correct answers for the whole of the training data is given in Figure 10. The dashed line gives the actual distribution for the first generation. Again each generation is subject to different sampling noise.

2) Compiling and Testing the Evolving Kernels: Unlike our earlier experiments with evolving DNA regular expressions [Langdon et al., 2010a] the time to compile the CUDA code dominates the fitness evaluation process. ([Harding and Banzhaf, 2009] describes a way of reducing the compilation overhead by running the compiler in parallel across a cluster of workstations.) There is some initial overhead with starting the compiler. It appears that the compilation time grows nonlinearly as the volume of code increases. Initial experiments suggested that the a good compromised between startup overhead and non-linear increase with size was when about

```
<intvar1> | <intvar2>
<intvar>
                  ::=
                               "pout" | "num_elements"
<intvar1>
                 ::=
                                "pin" | "offset"
<intvar2>
                 ::=
                         <intconst1> | <intconst2>
<intconst>
                 ::=
<intconst1>
                  ::=
                      <intconst1.1> | <intconst1.2>
                                  "0" | "258"
<intconst1.1>
                 ::=
                                  "1" | "strstart2"
<intconst1.2>
                 ::=
                                  "2" | "strstart1"
<intconst2>
                  ::=
                          <intexpr2> | <intspecialexpr>
<intexpr>
                  ::=
                            <intvar> | <intconst>
<intexpr2>
                  ::=
                       <intspecial1> | <line17.1.1.1.1>
<intspecialexpr> ::=
                  ::= <line8.1.1.1.1> | <line15.1.1.1.1>
<intspecial1>
                            <intop1> | <intop2>
<intop>
                  ::=
                          <intop1.1> | <intop1.2>
<intop1>
                  ::=
                          <intop2.1> | <intop2.2>
<intop2>
                  ::=
                                 "+" | "-"
<intop1.1>
                 ::=
                                  " * " | " >> "
<intop1.2>
                                 "<<" | "+"
<intop2.1>
                  ::=
                                  "-" | " * "
<intop2.2>
                  ::=
                  ::= "G_idata(" <strstart> <thidexpr0> ")"
<uchexprT>
                          <thidexpr> | <forintexpr>
<thidexpr0>
                  ::=
                          <uchexpr1> | <uchconst>
<uchexpr>
                  ::=
                        <uchexpr1.1> | <uchexpr1.2>
<uchexpr1>
                  ::=
<uchexpr1.1>
                  ::= "G_idata(" <intexpr> ")"
<uchexpr1.2>
                 ::= <uchexprT>
                 ::= "0"
<uchconst>
                  ::= <forcompexpr1> | <compexpr>
<forcompexpr>
<forcompexpr1>
                        <forintcomp> | <foruchcomp>
                 ::=
                 ::= <forintexpr> <compare> <forintexpr>
<forintcomp>
                 ::= <forintexpr1> | <intexpr>
<forintexpr>
<forintexpr1>
                 ::= <intexpr> <plusminus> "offset"
<foruchcomp>
                 ::= <foruchexpr> <compare> <foruchexpr>
                  ::= <foruchexpr1> | <foruchexpr2>
<foruchexpr>
<foruchexpr1>
                  ::= "G_idata(" <thidexpr0> ")"
<foruchexpr2>
                  ::= <uchexprT>
<compexpr>
                           <intcomp> | <uchcomp>
                  ::=
                          <intcomp1> | <intexpr>
<intcomp>
                  ::=
<intcomp1>
                  ::= <intexpr> <compare> <intexpr>
<uchcomp>
                  ::= <uchexpr> <compare> <uchexpr>
                                 "+=" | "="
<modassign>
                  ::=
                                  "+" | "-"
<plusminus>
                  ::=
                          <compare0> | <compare1>
<compare>
                  ::=
<compare0>
                  ::=
                         <compare00> | <compare01>
                                  "<" | "<="
<compare00>
                  ::=
                                 "==" | "!="
<compare01>
                  ::=
<compare1>
                  ::=
                     <compare10>
                                 ">=" | ">"
<compare10>
                  ::=
```

Fig. 8. Grammar int and uch expressions, comparisons and constants. Note the reuse of sub-expressions take from lines 15 and 17.

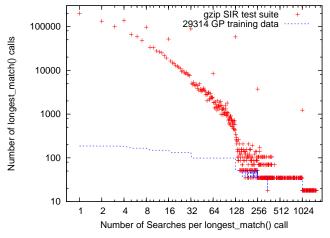


Fig. 9. Distribution of number strings gzip searches, for each byte compressed (211 SIR tests, total 1 599 028). The longest hash chain contains 1588 strings to be checked.

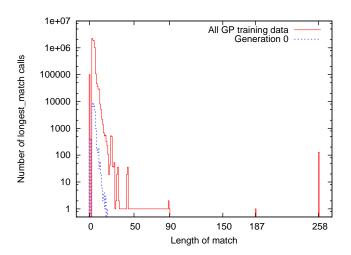


Fig. 10. Distribution of length of identical strings found by gzip when compressing 211 SIR files. (Due to interaction with gzip's hashing algorithm there are only 407 strings of a single byte and none with two bytes. In contrast there are 100 333 searches where the strings don't match at all and 2 222 039 where only their first three bytes are identical. The lower line shows a random sample used to train the GP in one generation.

one hundred kernels were compiled together. Therefore the gawk scripts which perform crossover and mutation also split the population into ten parcels each containing one hundred GP individuals.

Each parcel contains a 100-way switch statement and code which unpacks the arguments supplied by the host PC and directs the overall kernel to the right GP individual and returns its answer to the host PC. The ten parcels are compiled by the CUDA 2.3 nvcc compiler into ten object files which are linked into a single executable. (A slight reduction in compilation time, perhaps 18%, could have been obtained by moving the switch statement from the GPU .cu code to the host C++ code.)

The executable runs on the host Linux computer. It reads files holding the test data. Using gzip code, it calculates the answers that should be returned. (This is done only once.)

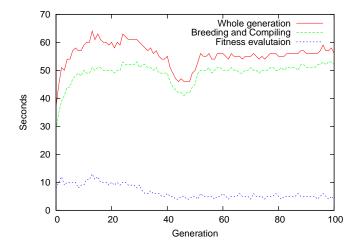


Fig. 11. Elapse time per generation.

For efficiency the whole GP population is run on each test example and their answers saved. This allows the data needed for each training example, which typically includes 64kb of data from the file to be compressed, to be loaded only once for all 1000 members of the population.

As mentioned above, the population is compiled into ten object files. For the purposes of training the GP, each is a separate CUDA kernel. The host launches these ten kernels in series. Their answers are returned to the host before the next is launched. In principle, this could be done in parallel. Each of the ten kernels runs its one hundred GP individuals across the whole of the current hash chain in parallel. While the exact number of parallel execution threads running on the GPU will vary between each of the one hundred training examples, the mean hash chain length is 293 meaning on average the GPU runs $293 \times 100 = 29300$ parallel operations. For each generation it will do 10 kernel launches for each of the 100 training examples. On average over the first 100 generation fitness testing takes 6.5 seconds per generation (which includes host file processing). In contrast generating and compiling the population takes 49.0 seconds (most of which is in the nvcc compiler). See Figure 11

3) Error based Fitness and Penalties: For each test case the GPU will return up to 1000×1588 answers. Each is one of the population's answer to the question how many bytes match between two points in the current 64kb window onto the file being compressed. As described in Section III-B.2, we already know gzip's answer to this question. This allows us to define an error based fitness function. Taking care of overflow, fitness is essentially the sum of the absolute error between the answer returned by each GP and that calculated by the gzip code. N.B. in each test, each member of the population's fitness could be increased (i.e. worsened) up to 1588 times.

Looking again at the blue dashed line in Figure 10, we see some values are much more common in the training data than others. A simple strategy for obtaining a good fitness is to choose one of these and always return this constant value.

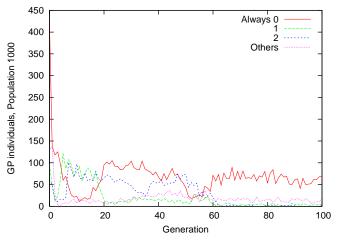


Fig. 12. In the initial population 70.7% of random CUDA kernels return a fixed value. This eventually settles near 6.9%, most of which always return zero, despite the fact it carries the largest fitness penalty.

To deter this and limit the number of children allocated to constants a penalty of 10 000 times the number of training examples was introduced. (E.g. in the initial generation the 100 fitness cases together cover 29 328 putative string matches. This means the minimum constant penalty is 293 280 000.) The penalty is is doubled if the constant was 2, tripled if it was 1 and quadrupled if it was 0. Figure 12 plots the evolution of the number of GP individuals in the population which, despite these penalties, effectively ignore their inputs and always returns the same answer.

4) Debug: There were many troubles during the development process. In the end it was decided to sacrifice the last member of the GP population and repace it was a non-evolved code fragment which simply reflected the training data. This allowed the host code to verify that the training data had indeed been copied successfully to the GeForce 295 GTX nVidia GPU graphics card, the kernel had been run and had been able to read its inputs, and it had succeeded in returning its answer to the host. If any of these steps fail, the Unix process is aborted until the problem is manually resolved.

IV. EVOLVING SOLUTIONS

The details of our grammar based GP are given in Table I. The terminal and function sets are determined by the grammar described in Section III-A. The error based fitness function was described in the previous section.

The evolution of the population in the first run is plotted in Figure 13. The first solution was found in generation 50. Figure 14 shows the first 100 generations of the evolution of all ten runs. Five runs out of ten reported solutions by generation 100. (I.e. the chance of success is highly signficant (p=0.01) better than one in seven.)

Given the BNF grammar all that is needed to construct each program is the sequence of choices made at the binary alternative nodes. The rest of the parse tree and hence the program itself can be inferred from these. If we treat the

TABLE I
STRONGLY TYPED GENETIC PROGRAMMING PARAMETERS FOR
AUTOMATICALLY CREATING A CUDA KERNEL.

1	Function:	Binary rules in the BNF grammar (Figures 2–8).
-	Terminals:	Grammar rules without alternatives (Figures 2–8). Array
		g_idata indexes forced to legal range by reducing modulo
		array size.
]	Fitness:	Sum of absolute errors between evolved answer and that
		given by grin coross pseudo random sample of 100 of

sum of absolute errors between evolved answer and that given by gzip across pseudo random sample of 100 of 29 315 examples drawn from compressing files in the SIR test suite (Section III-B.1).

Selection: Generational. 4 members tournaments. New sample of

training data each generation.

Population: 1000

Initial pop: Ramped half-and-half 2:6 Parameters: 50% subtree crossover, 5

s: 50% subtree crossover, 50% subtree mutation. Crossover and mutation points are chosen uniformly (i.e. without a function bias [Koza, 1992]) treating each grammar rule as a distinct type and ensuring each offspring is genetically different from both its parents. Max depth 100, No size limit. ok () (Figure 2) limits all for loops to 2000

iterations.

Termination: 1000 generations

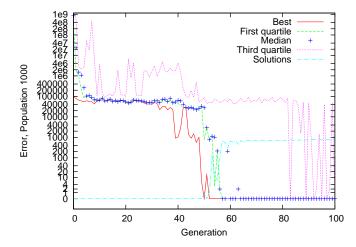


Fig. 13. Absolute error and constant penalty across $\approx 29\,000$ training examples (varies between generations). Note non-linear vertical scale. Typically after generation 57 about $\frac{3}{4}$ of the population pass all the tests.

parse tree as the important underlying skeleton of the evolved code and look at two statistics (the number of nodes in the tree and their maximum depth from the <start> node) we see the usual patterns of GP. Figure 15 shows bloat with the trees increasing in size and depth. When we look at the evolution of both size and depth (Figure 16) we see the same behaviour as traditional GP benchmarks (symbolic regression and the Boolean multiplexor) [Langdon, 2000]. It is remarkable that a strongly typed grammar based GP which highly constrains evolution to ensure syntactically correct, executable C++ code yields such similar behaviour to tree GP where crossover may occur between any two points in the parent trees. This is reinforced when we look at the evolution of the distribution of the number of choice nodes in the individuals. Again we see the characteristic "humped" distribution with a long upper tail, with both the location of

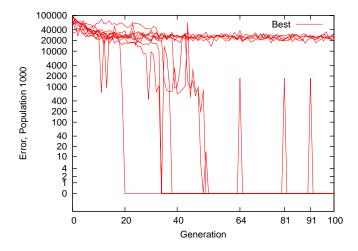


Fig. 14. Evolution of smallest error in the population in ten runs. Note in one of the five runs which found solutions (error=0) the solution was missing in generations 54 and 91. In another it was missing in generation 81. In the other three cases the solution was quickly locked into the population despite the varying fitness function and the absence of elitism.

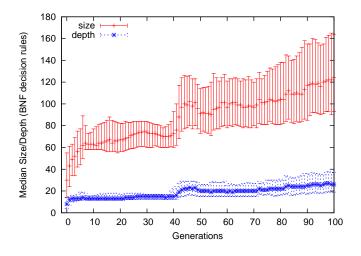


Fig. 15. Evolution of size and depth of BNF parse trees in first GP run. Error bars show inter quartile range.

the hump and tails increasing over time. (The distributions of the initial population and generation 100 are plotted in Figure 17.) [Dignum and Poli, 2007] shows (for a totally different GP system) that this is a Lagrange distribution. Note even in generation 100, when most of the population are highly fit, mutation is producing small unfit children.

Circular lattices [Daida et al., 2005] [Poli et al., 2008, p 136] are a nice alternative way to display GP trees. They emphasis the shape of the tree rather than its contents. (Contrast Figures 23 and 24 which both refer to the underlying parse tree of the same evolved CUDA kernel.) Figure 18 plots every GP individual in the initial population. The <start> rule is notionally the center of the circular display and we do indeed see its ten subsequent rules clustered around it. However some of these (e.g. e.g. ne2>) are always immediately expanded into terminals of the grammar and cannot be

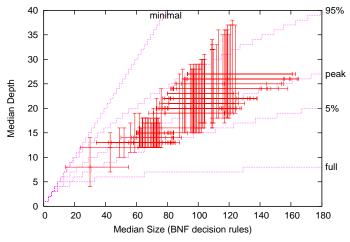


Fig. 16. Evolution of size and depth of BNF parse trees in first GP run (Medians and interquartile ranges, as Figure 15). Dotted lines indicate location of underlaying distribution of numbers of binary trees of a given size [Langdon, 2000]. Note high fitness evolved trees have the same shapes as random binary trees.

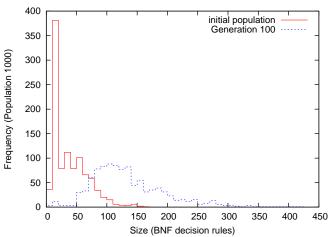


Fig. 17. Distributions of number of decision BNF rules in 1000 random CUDA gzip kernel parse trees and distribution after 100 generations of evolution. Note similarity with Lagrange distribution [Dignum and Poli, 2007].

expanded further. Other rules (e.g. <line10-20>) must be followed by other rules and large subtrees are often attached to them. Thus the rules of the BNF grammar give rise to asymmetric trees. However this only exacerbates the natural tendency for half-and-half random initialisation to produce asymmetric trees.

The population also converges in the sense that the parse trees have similar shapes. Figure 19 shows the whole population after 100 generations. Again the strongly typed GP trees behave as would be expected of an ordinary tree based GP.

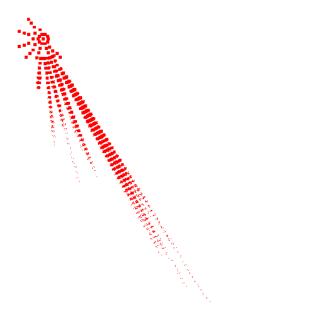


Fig. 18. 1000 parse trees for random initial population displayed as a circular lattices [Poli *et al.*, 2008, p 136] [Daida *et al.*, 2005]. (Noise used to spread data.)

V. VALIDATING THE SOLUTIONS

The evolved kernel (Figure 20) passes all the tests in generation 50. It does what it was evolved to: run on the graphics card and provide answers that emulate those needed by gzip.

Notice that although it skips the second byte, in its context in gzip and in particular the way gzip's hashing works, this is very unlikely to make any difference. Indeed when the evolved kernel is compiled as part of gzip's longest_match routine the new code produces identical answers. This was verified in a version of gzip containing both old and evolved code and compressing gzip_1.4.tar (19 322 880 bytes) which contains the complete release for SIR including the whole SIR test suite. However it might, in principle be possible to find an example where it fails.

On average only every fifth generation contains training examples with strings which match for exactly one byte (see Figure 21). The first such generation after the first solution has been evolved is generation 55. Generation 55 contains seven programs which pass all its tests. The last one was chosen as being the simplest to explain. It is given in Figure 22. The corresponding grammar parse tree is given in Figures 23 and 24. Notice it checks all the bytes in both strings. Like the solution evolved in Generation 50 it succeeds on all gzip_1.4.tar (19 322 880 bytes). It also passes contrived tests designed to check for one byte errors run outside gzip.

Obviously such naive kernels do not provide any speed up.

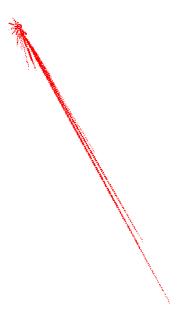


Fig. 19. 1000 highly evolved trees from generation 100 of the first GP run. (Scale 2.6 times that of Figure 18 Again noise used to spread data.)

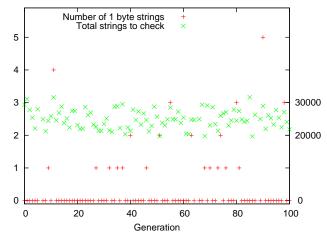


Fig. 21. There are only 407 cases where training strings match for exactly one byte (cf. Figure 10). This means they are unlikely to be selected at random. Indeed they are found only in 20 generations in the first $100 \ (+)$. The total number of strings used for training in each generation is plotted with \times . Note change of scales.

VI. DISCUSSION

gzip is a venerable Unix utility which still betrays its roots in 16-bit computing. It has been highly optimised and, in particular, the string match finding code has been carefully contorted by skilled human programmers to get the best from traditional single CPU systems. It contains several heuristics which are designed to give a good tradeoff between execution time, and degree of compression obtained on typical ascii Unix text files. Doubtless better tradeoffs could be obtained for other circumstances. For example, where the files to be compressed are specialised or when running on new hardware (such as remote robotic platforms) where energy consumption rather than compression speed

```
__device__ int kernel538(const uch *g_idata, const int strstart1, const int strstart2)
{
  int thid = 0;
  int pout = 0;
  int offset = 0;
  int num_elements = 258;
  for (offset = 2; G_idata( strstart1+ pin ) == G_idata( strstart2+ pin ) ;offset ++ )
{
  if(!ok()) break;
  {{    pin = offset;
    thid = G_idata( strstart2+ pin*num_elements+thid - offset);
  }}  pout = 1 - pout;
}
return pin;
}
```

Fig. 20. gzip CUDA kernel evolved in generation 50. (Bold {} delimit for loop body.) In the context of gzip's longest_match the fact that it returns 2 rather than 1 when there is only a single byte in common between strstart1 and strstart2 is very unlikely to matter since both are too short for gzip to use in its compression routines. It is computationally easy to do the inverse operation (double check the longest match is indeed a match) compared to searching the file looking for the longest match. (gzip pragmatically may stop looking for the optimum match and, for efficiency, accept one that is good enough.)

```
__device__ int kernel978(const uch *g_idata, const int strstart1, const int strstart2)
{
  int thid = 0;
  int pout = 0;
  int offset = 0;
  int num_elements = 258;
  for (offset = 1; G_idata( strstart1+ pin ) == G_idata( strstart2+ pin ); offset ++)
{
  if(!ok()) break;
  thid = G_idata( strstart2+ thid );
    pin = offset;
}
return pin;
}
```

Fig. 22. gzip CUDA kernel evolved in generation 55. Notice that apart from the correct initial value for offset in the for loop (bold {} delimit loop body) and the irrelevant assignments to thid, it is identical to solution found five generations earlier (cf. Figure 20).

is paramount. Relaxing limitations inherited from its 16-bit heritage might also allow different tradeoffs between memory used by gzip, CPU time and size of compressed output. All of these involve a small fraction of gzip which is heavily used. Existing profiling tools can automatically identify such regions in much bigger legacy systems.

Although a pure approach might require total automation, it may be that a pragmatic approach which allows evolution to try novel "outside-the-box" tradeoffs between complex objectives and hoping the Pareto optimal evolved solutions will inspire human engineers to create and field a trusted interface which wins in the long run.

We have used gzip as a demonstration system for an approach which seeks not to re-engineer complete systems but to automatically generate code for tiny fraction of them. Given an existing system and its test suite. We have the bare bones to evolve a replacement for part of it. The existing system can be used as the "Gold standard" for the desired functionality. There is no need to creat a formal specification. Each trial replacement can be simply compared with the existing system. We have taken the approach of requiring the replacement code to run on new hardware but there might be other, indeed multiple, objectives for the new plugin.

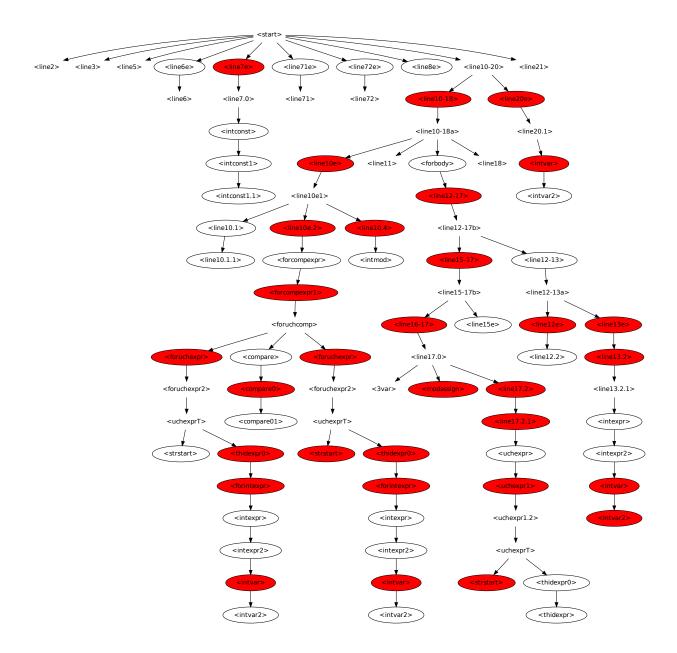


Fig. 23. Grammar for gzip CUDA kernel evolved in generation 55. (cf. Figure 22). Ovals indicate binary decision rules. With shaded ovals the second option was used.

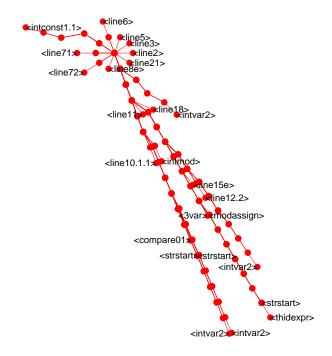


Fig. 24. Grammar for gzip CUDA kernel evolved in generation 55 (cf. Figure 23) displayed at a circular lattice [Poli *et al.*, 2008, p 136] [Daida *et al.*, 2005]. The last rule in each branch of the BNF grammar tree (Figure 23) is labeled.

For efficiency gzip deliberately relaxes its own objective of finding the longest match between strings in a file in a number of ways. So that it sometimes uses a match even though it is not guaranteed to be the longest. However although the search may have taken a long time it is relatively quick to verify the reported match is indeed a match. Indeed the debug version of gzip, has such a check permanently enabled. It is not uncommon for operations which are expensive in the forward direction to have a (relatively) cheap inverse check, or sanity check. E.g. after performing statistical calculations (mean, standard deviation) on large numbers, it is cheap to double check that their variance is non-negative.

We have adapted [Weimer *et al.*, 2009]'s approach, and used the manufacturer's supplied code as a template to guide the evolutionary process. We can see that in the general case there will always be the existing system to use as a guide. Whilst nVidia supplied only 67 modestly sized codes the volume of code typically available is much bigger. Even so such code may not be too large to prevent automatic analysis. Indeed this may be the next aspect of GYPSE we seek to demonstrate.

Whilst the grammar is firmly based on the nVidia supplied example, we have incorporated little information from gzip. Another approach to be tried in the future might be to investigate more automated ways of combining information gleaned from multiple sources.

VII. CONCLUSIONS

Automatic programming has been a goal of Computer Science for many years. With millions of people employed programming computers this still seems very remote. Nevertheless by pragmatically adopting less lofty ambitions progress is being made in automating software engineering and particularly software maintenance [Arcuri and Yao, 2008; Weimer *et al.*, 2009]. The long-term goal of GIPSE is to automate the construction of small quantities of high value code (rather than to create whole systems). Particularly either code which is critical to the efficiency of the whole program or is critical to the success of a new system by linking together traditional codes. We have started with a small but achievable goal, where we have taken a crucial part of a venerable C program and GIPSE has ported it to new hardware.

Whilst automatic bug fixing has been demonstrated on a number of real C and C++ codes, this is the first time computational intelligence has been used to automatically create an nVidia CUDA kernel which runs in parallel on a state of the art graphics card as part of a legacy system. Section II described the context free grammar created from example code supplied by nVidia, whilst Section III gave full details of the strongly typed grammar based genetic programming (GP) system. The results in Section IV show it is possible to evolve C++ code. The automatically created kernels have been tested (Section V) back-to-back with the original code many millions of times without error.

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