

# Long-Term Stability of Genetic Programming Landscapes

Workshop on [Landscape-Aware Heuristic Search](#), 16 July 2017, Nadarajen Veerapen *et al.*, GECCO-2017, Berlin, Germany. Room Diamant

[W. B. Langdon](#)

Department of Computer Science

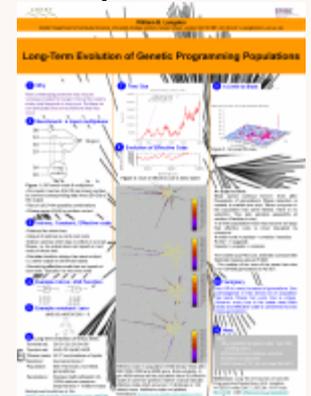


Long-Term Evolution of GP Populations  
Poster Monday 17:50-20:00  
GECCO companion p235-236

Technical report RN/17/05  
<https://arxiv.org/abs/1703.08481>

[Video1](#)

[Video2](#)



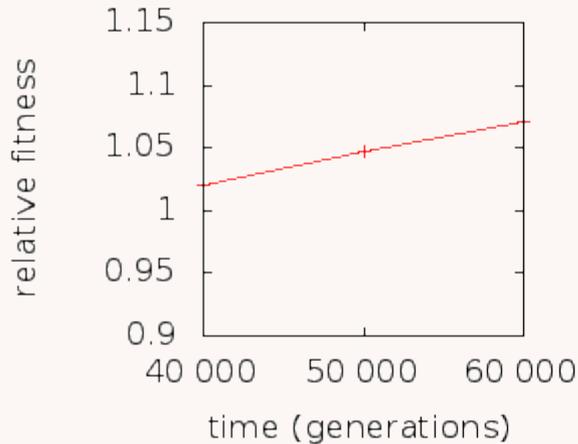
# Genetic Programming and Long-Term Evolution Experiments

- Why we care about LTEE
- Evolving Bacteria 60,000 generations v. evolving programs 100,000 generations
- LTEE continuous innovation v convergence
- Existing results on landscape of large trees
- New results
  - Increase in code (bloat), end of bloat
  - Theory some true, some less so
  - Evolution has smoothed large tree landscape although fitness distribution remains rugged.

# Why Long Term Evolution Matters

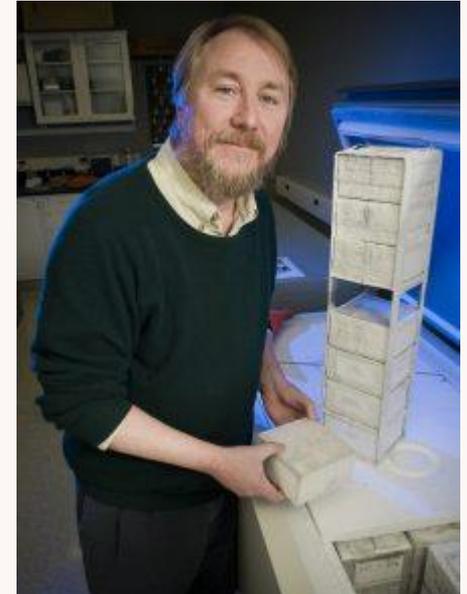
- More challenging problems may require running evolution for longer.
- Hence the need to study what happens in long runs.
- By mapping landscape far from origin, perhaps we can anticipate and solve problems that may occur.

# Long-Term Evolution Experiment



Mean fitness of nine *E. coli* populations from the LTEE

Evolving Bacteria 60,000 generations  
Even after 60000 gens fitness still improving



Richard Lenski pulls frozen bacteria cultures out of a freezer 15 Oct 2009

R. E. Lenski *et al.* 2015. [Sustained fitness gains and variability in fitness trajectories in the long-term evolution experiment with \*Escherichia coli\*](#). Proc. Royal Soc.

# Convergence of Fitness Distribution

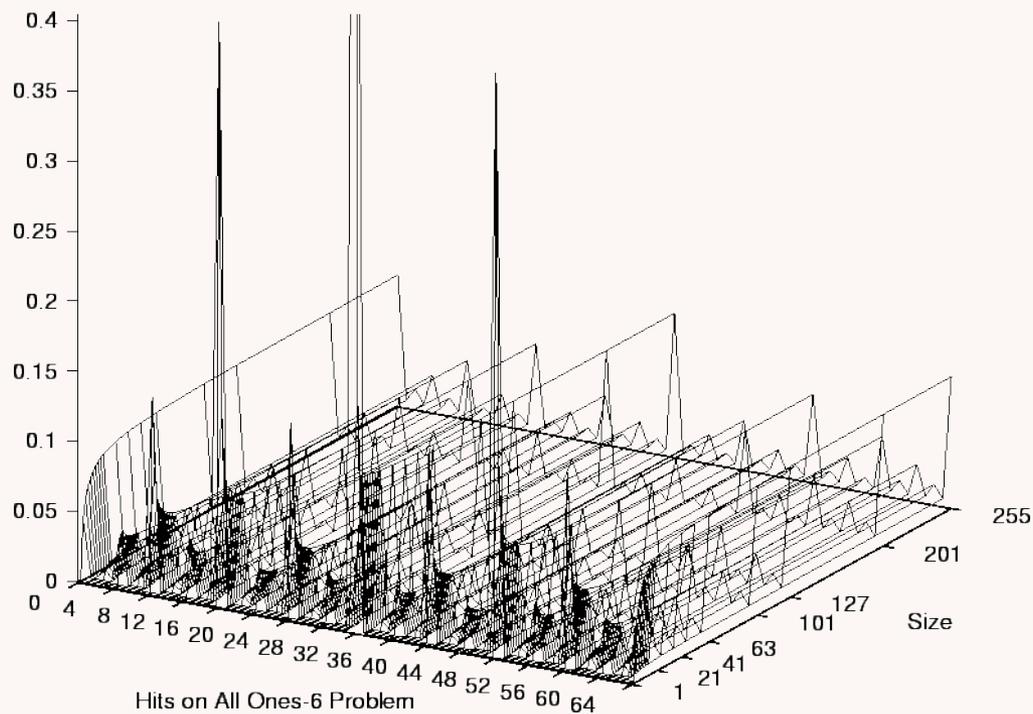


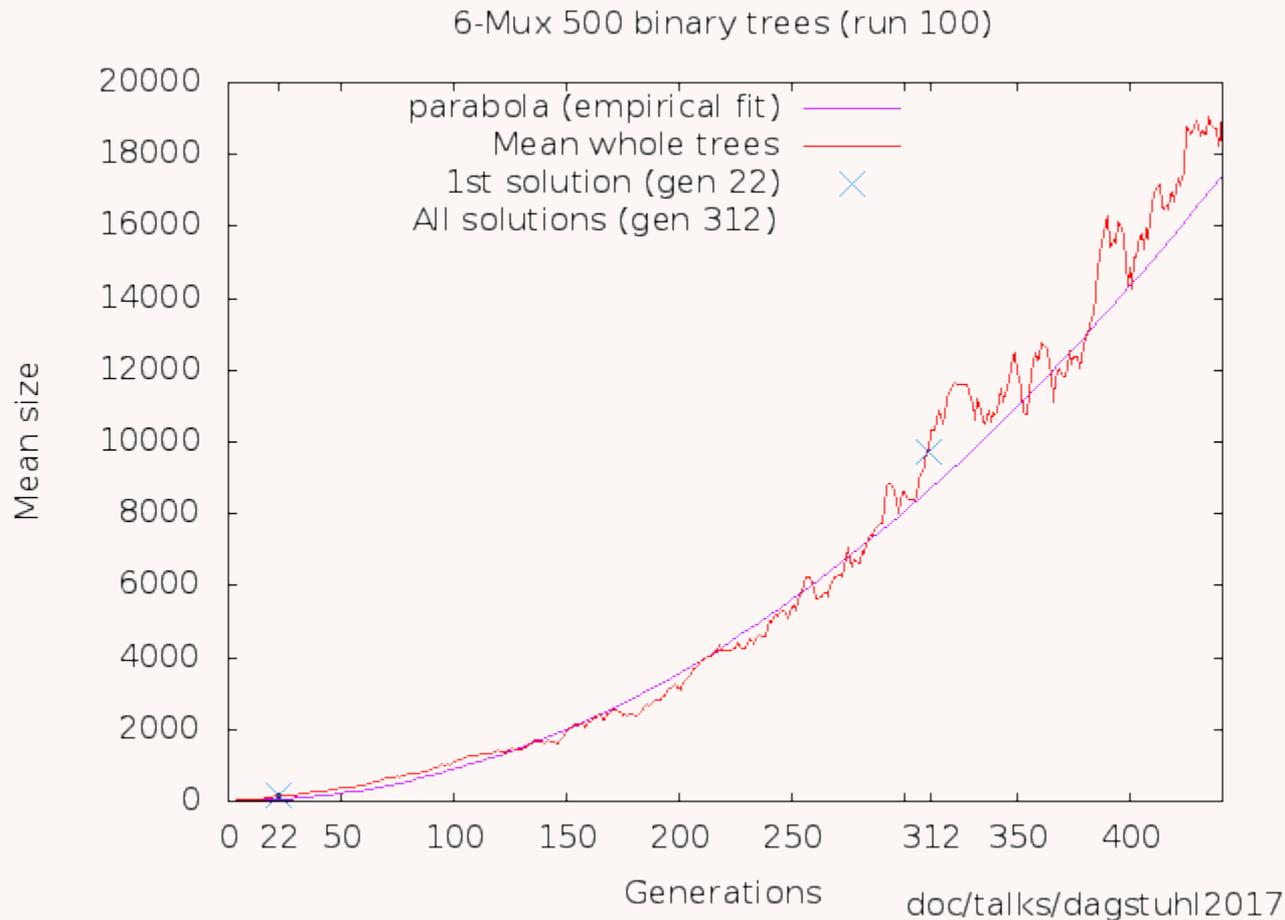
Fig. 7.7 [Foundations of Genetic Programming](#)

Large tree fitness distribution remains rugged like small trees but as we will see GP population maintains smooth landscape

# Genetic Programming and Long-Term Evolution Experiments

- GP system able to run thousands of generations. (Do not stop when solved)
  - Expect bloat (tree growth)
  - Compact representation of trees
  - Fast fitness evaluation
    - GPquick C++, written by Andy Singleton
    - ≈ two bytes per tree node
- Submachine code genetic programming

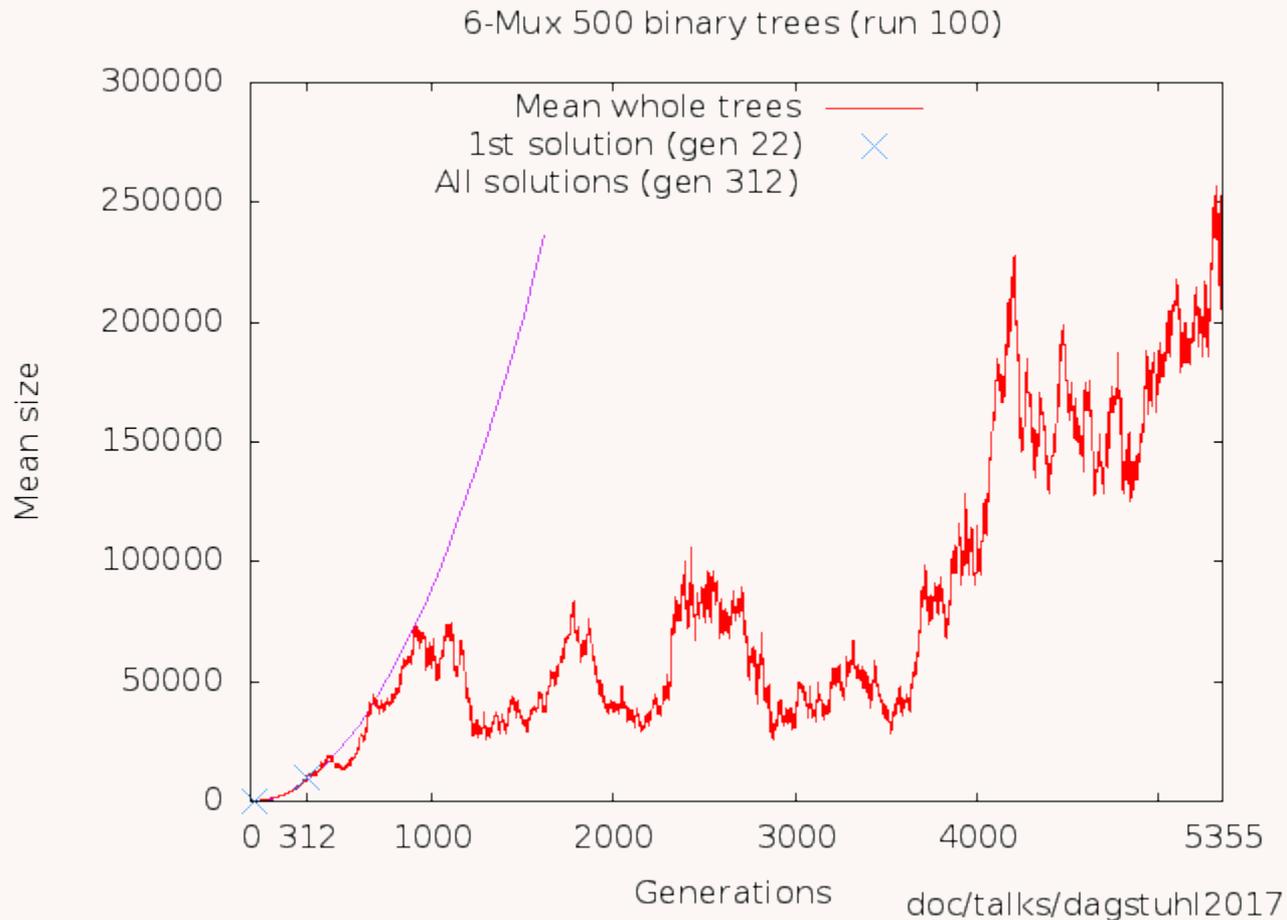
# Evolution of Program Size



Note evolution continues after 1<sup>st</sup> solution found in generation 22 and even after 1<sup>st</sup> population when everyone has maximum fitness (generation 312).

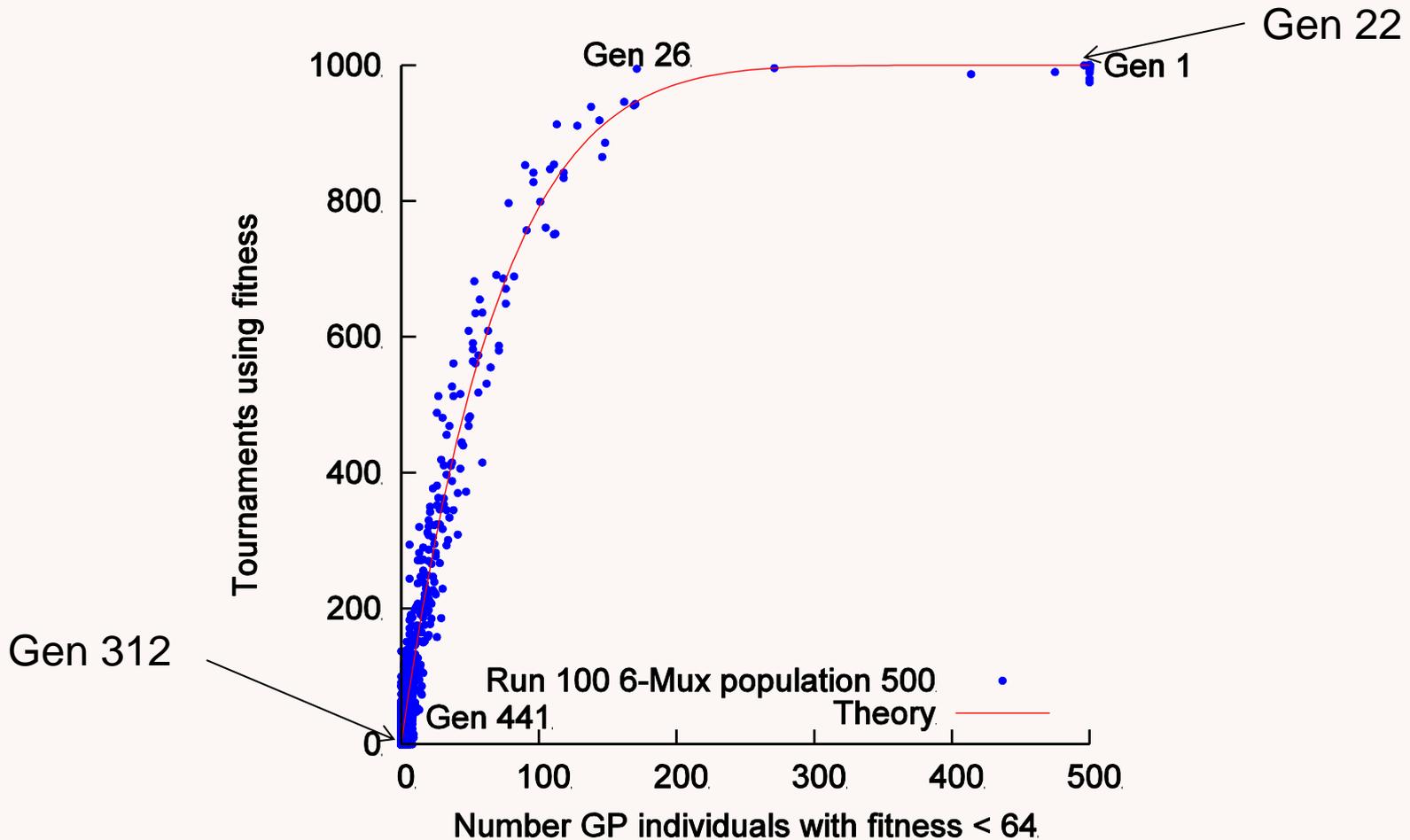
[GP+EM \(1\)1 pp95-119](#)

# Evolution of Program Size



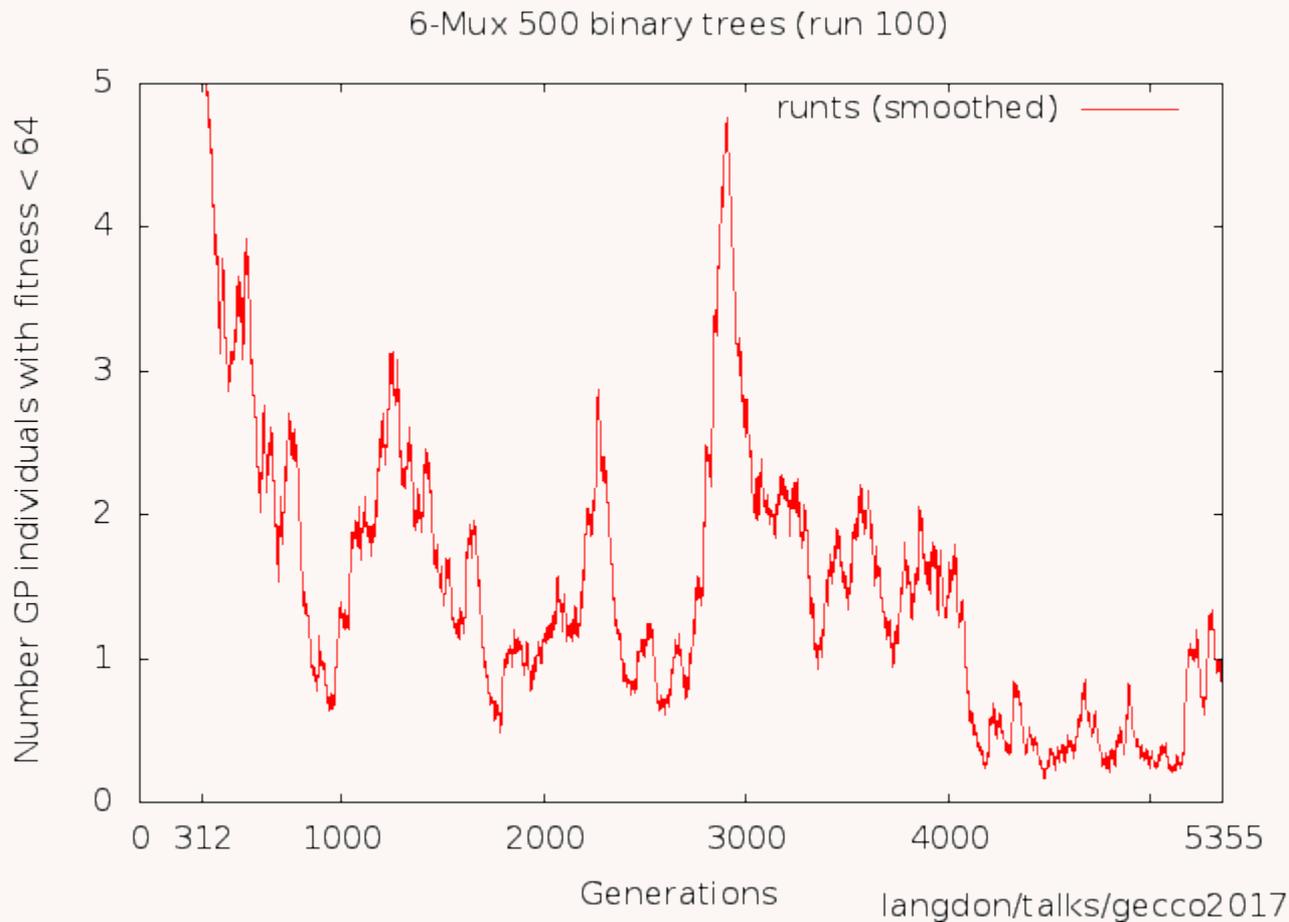
Note evolution continues even after 1<sup>st</sup> population when everyone has maximum fitness (generation 312) but tree size falls as well as rises.

# 6-Mux Fitness Convergence



Theory  $y = 2\text{popsize}(1-(1-x/\text{popsize})^7)$  matches experiment

# 6-Mux Fitness Convergence



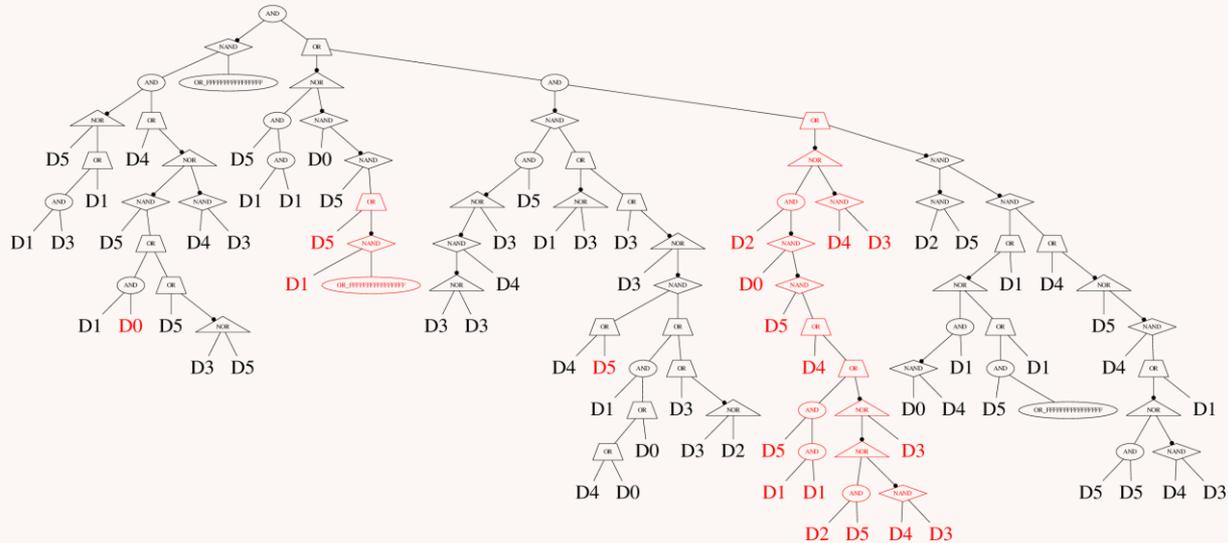
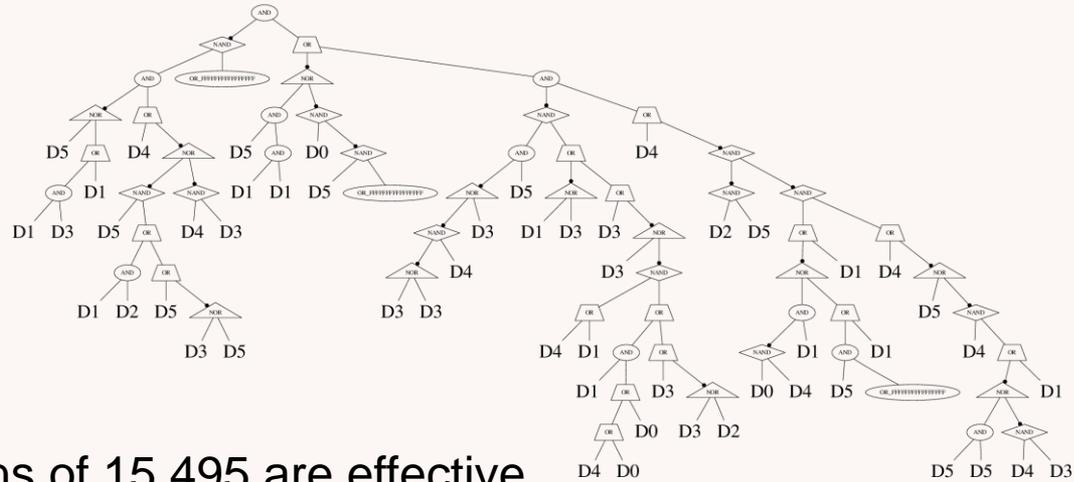
Plot smoothed by taking running average over 30 generations

# CREST Convergence in Genetic Programming

- GP genotypes do not converge. Even after many generations every tree in the population is different, BUT...
- Every (or almost all) trees give the same answers (phenotypic convergence)
- Effective code, i.e. code to solve problem, does converge.

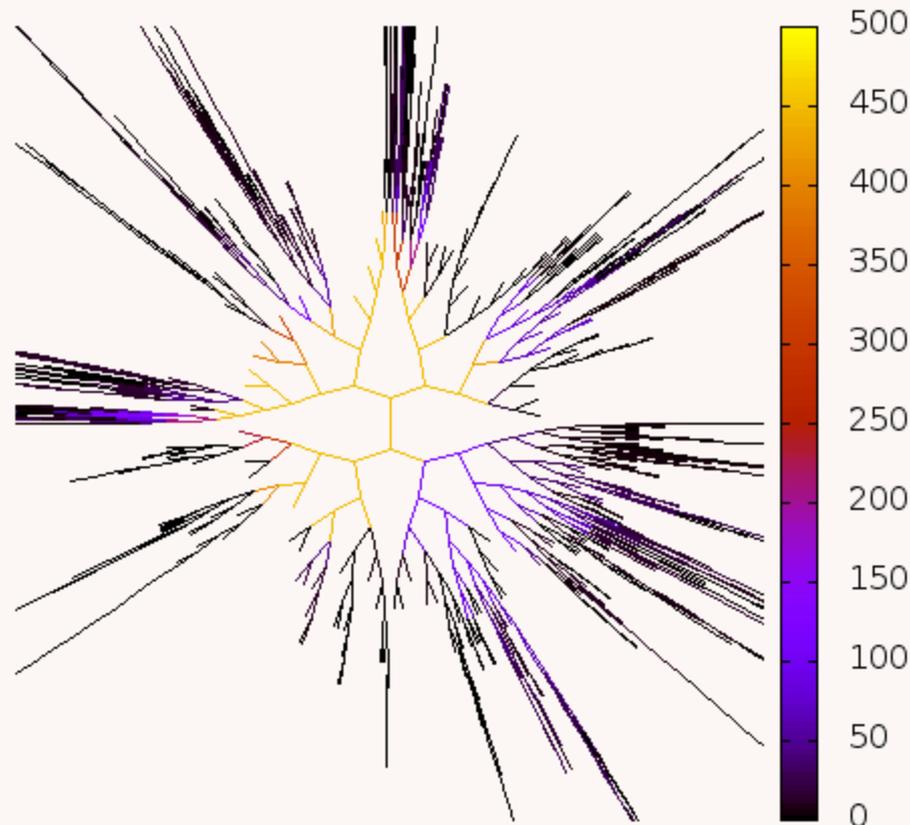
Effective code other runs converges differently

# Convergence of typical Effective Code

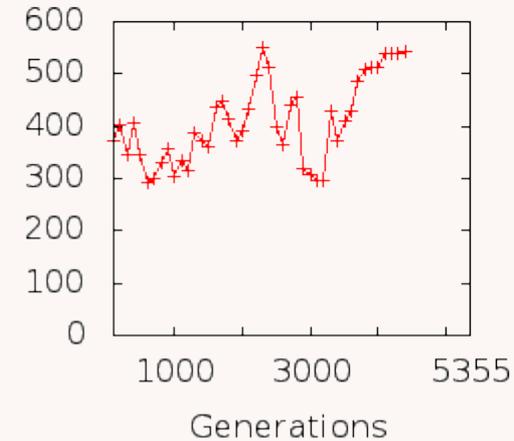


# Convergence of Effective Code

6-Mux (binary tree) population at generation 100



Mean effective code  
(run 100, population 500)



wlangdon/gp/eden mux/4-jun-2016 100

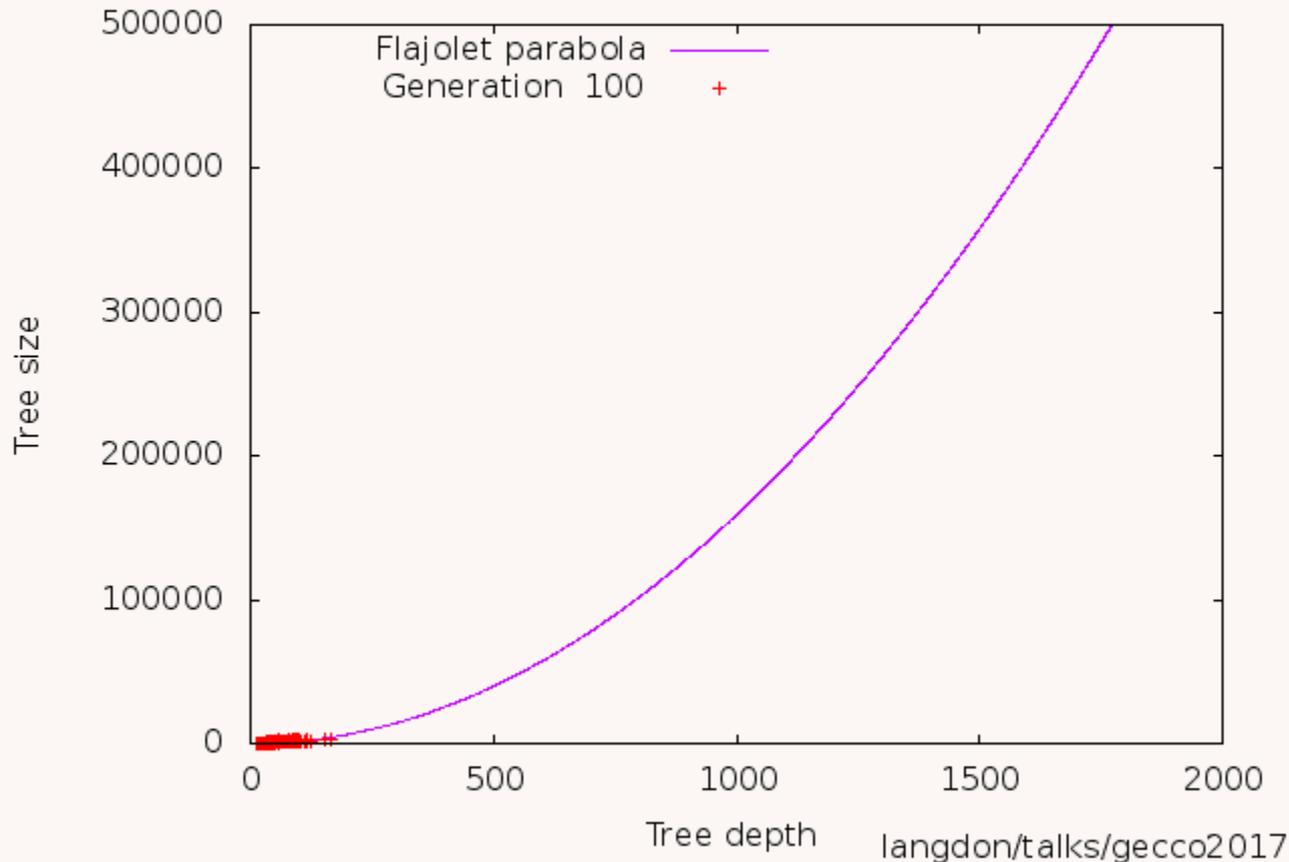
[Video](#)

Effective code only. Yellow highly converged.  
Black unique code

Circular lattice code [gp2lattice.awk](#)

# Shapes of Evolved Trees

6-Mux 500 binary trees (run 100)



[Video](#)

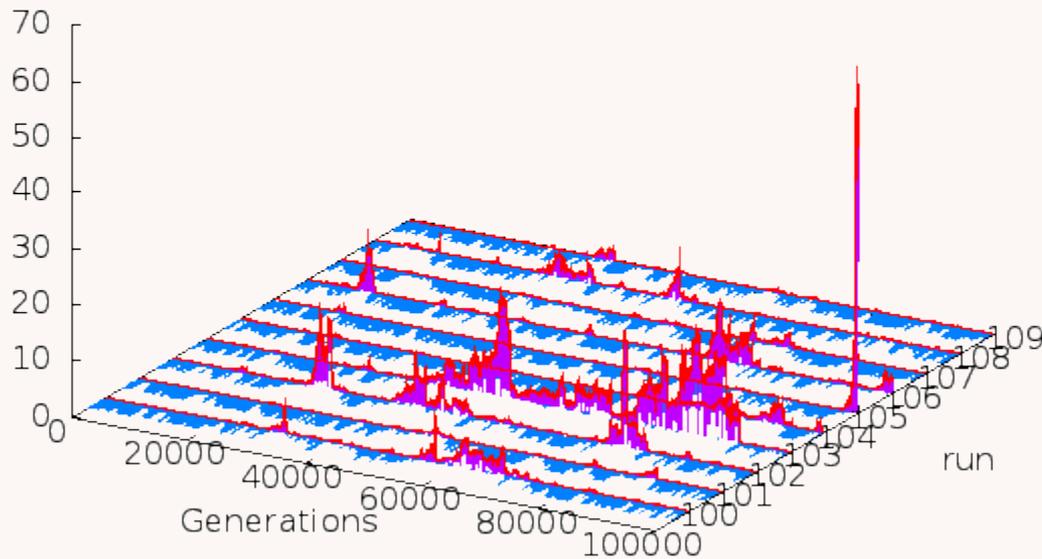
Both whole trees **+** and subtrees lie near  
Flajolet  $\text{Depth} \approx 2 \left( \frac{\pi \text{size}}{2} \right)^{1/2}$  limit<sup>1</sup> for random trees

# Bloat limited by Gambler's ruin

- Tiny fraction of disrupted (low fitness) children sufficient to drive evolution towards ever bigger trees.
- As trees get bigger chance of hitting protected effective code near root node falls.
- In a finite population eventually no child will be disrupted.
- Size, without fitness, wanders at random.
- But wandering towards lower limit will re-establish the conditions for bloat.  
cf. Gambler's ruin.
- Very approximate limit on tree size:  
tree size  $\approx$  number of trees  $\times$  core code size

# Bloat limited by Gambler's ruin

Mean size (millions). Ten runs, population 50 trees

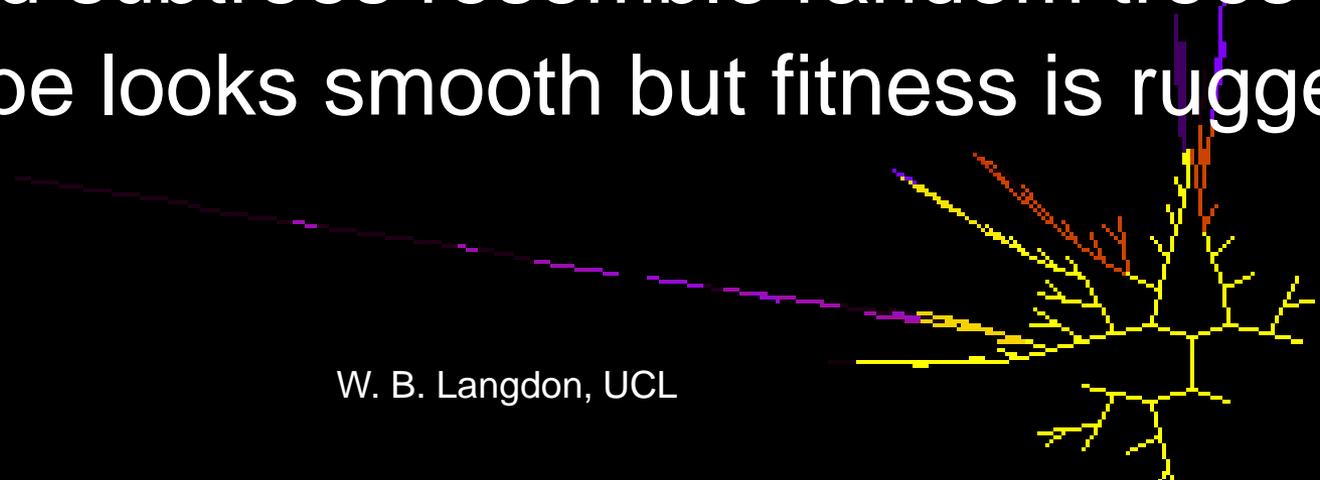


- tree size  $\approx$  number of trees  $\times$  core code size
- tree size  $\approx 50 \times 497 \approx 25\,000$
- Across ten runs and 100,000 generation, median mean size **42 507** (smallest tree in pop size=10 513)

In all ten runs the whole population repeatedly collapses towards smaller trees

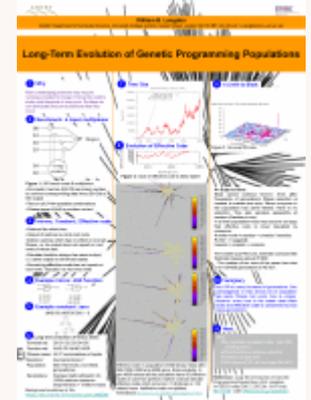
# Conclusions

- Studied long term evolution ( $\gg$ any other GP)
- 100s gens where everyone has same fitness
- No selection to drive size increase
- Gambler's ruin with size falling as well as rising
- Evolved effective code surrounded by ring of sacrificial constants and introns
- Trees and subtrees resemble random trees
- Landscape looks smooth but fitness is rugged



Technical report RN/17/05  
<https://arxiv.org/abs/1703.08481>

END



<http://www.cs.ucl.ac.uk/staff/W.Langdon/>

<http://www.epsrc.ac.uk/> **EPSRC**

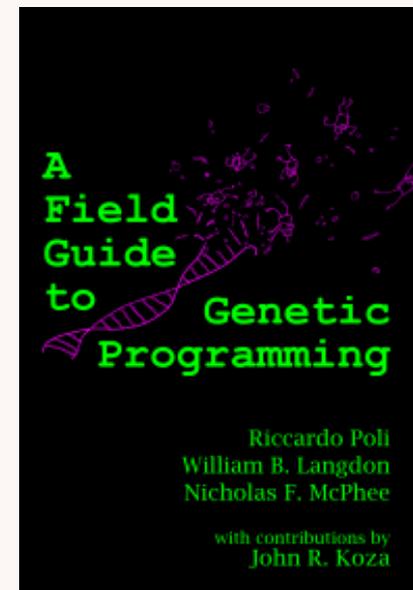
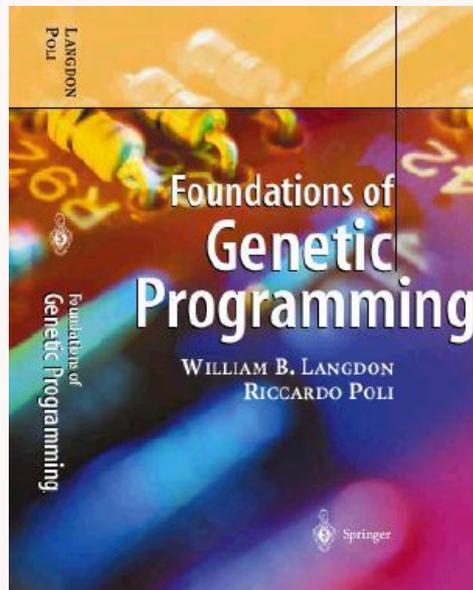
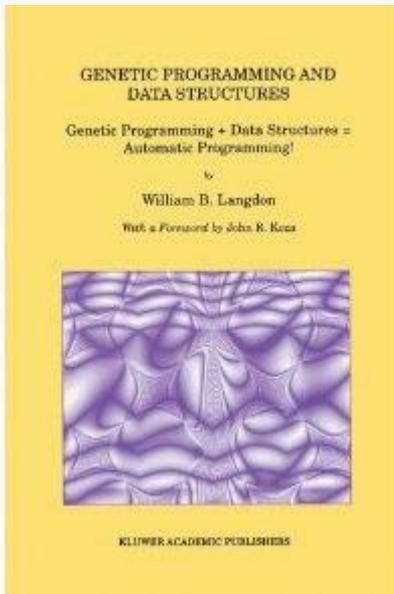
# Genetic Programming



W. B. Langdon

CREST

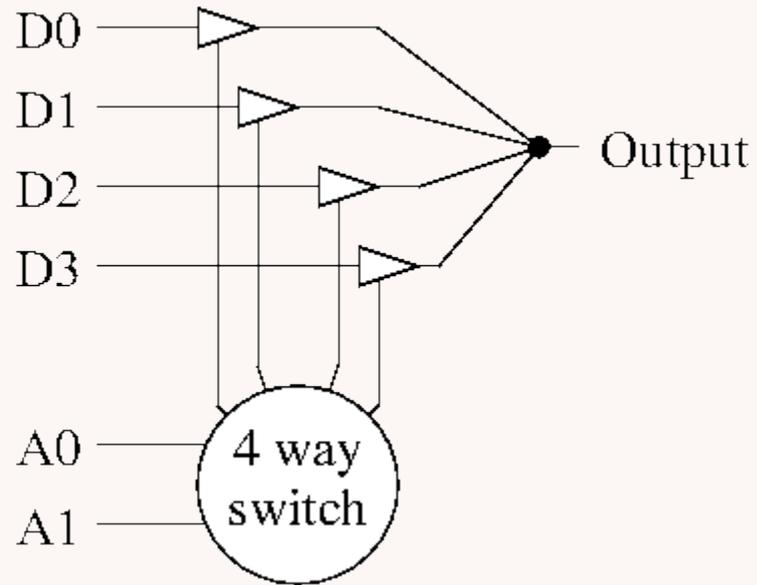
Department of Computer Science



# GPquick

- GPquick C++, written by Andy Singleton
  - ≈ two bytes per tree node
- Submachine code GP
  - Boolean (bit) problems.
  - AND, NAND, OR, NOR operate simultaneously in parallel on bits in word (e.g. 32 or 64 bits)
  - 64 bit computer can do 64 test cases in parallel

# 6 Multiplexor



- GP bench mark.
- Six inputs:
  - Use two (D4 D5) as binary number to connect corresponding data lines (D0-D3) to the output
- Test on all  $2^6=64$  possible combinations
- Fitness score (0-64) is number correct

# Genetic Programming to solve 6-Mux

- Terminals (tree leafs)
  - D0,D1,D2,D3 D4,D5
- Function set: 2 input gates → binary trees
  - AND, NAND, OR, NOR. No side effects
- Generational population of 500 trees
- Tournament selection: choose best of 7
- 100% subtree crossover
- Initially hard limit on tree size ( $10^6$ )

# Impact of Subtrees

- Subtree like whole tree.
- Output of subtree is via its root node
- **Intron**: subtree which has no effect on overall fitness. I.e. its output does not impact on root node of whole tree.
- **Constant** subtree always has same output, i.e. same output on all 64 test cases.
- Remaining **effective code** has an impact on root node. Typically it is next root node

# Example Intron: AND Function



A	B	Out
0	0	0
0	1	0
1	0	0
1	1	1



A	B	Out
0	0	0
0	1	0
1	0	0
1	1	0

Left: two input AND node.

Right: same but input B is always 0.

So output always 0. Input A has no effect.

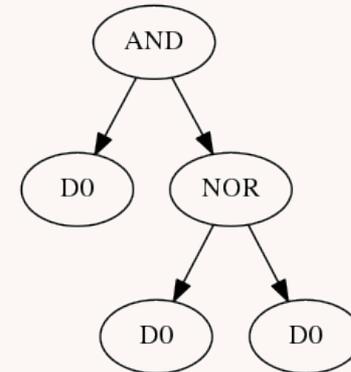
Subtree A is always ignored, even in child.

(NB no side effects)

# Constants

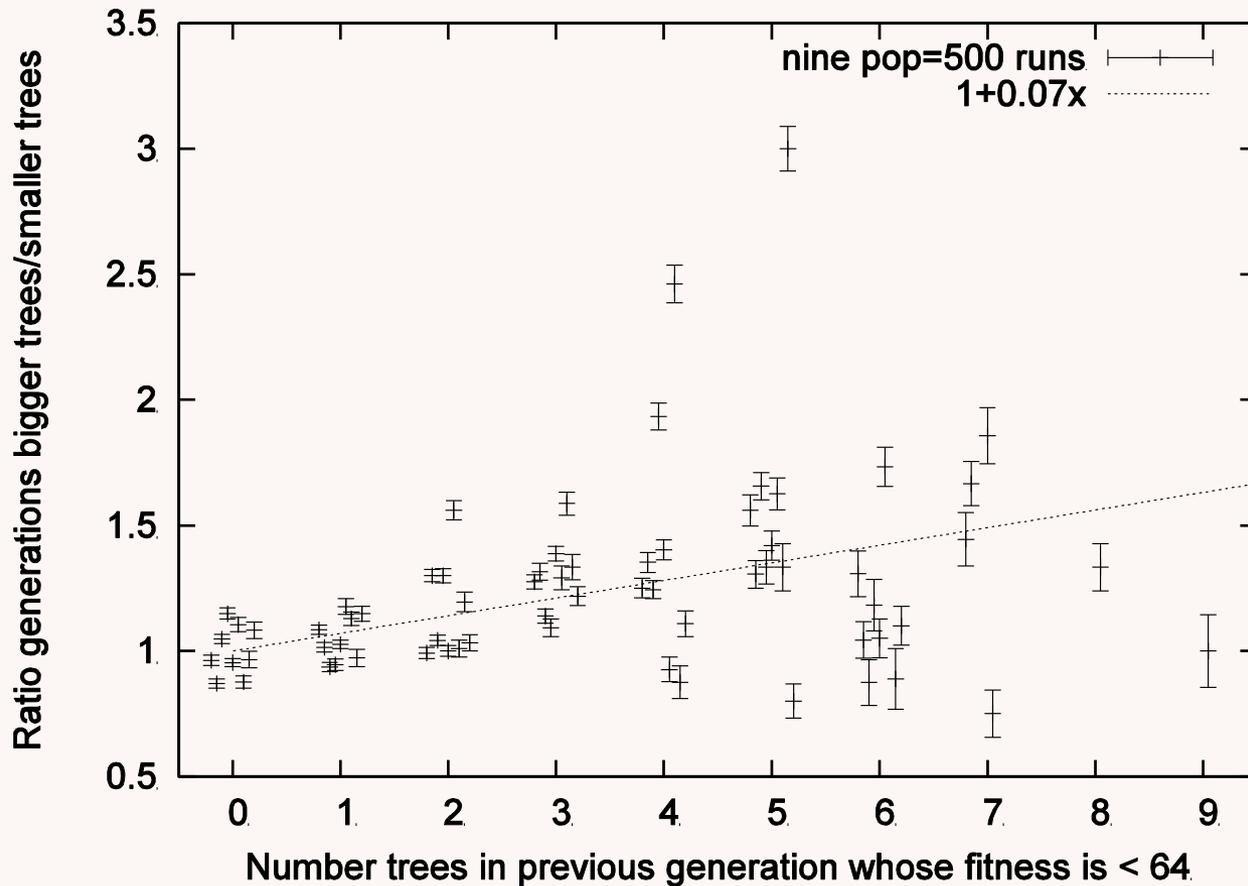
- Two constants: always 0 and always 1 (FFFFFFFFFFFFFFFF).
- E.g. evolve by negating input and ANDing with same input

$$(\text{AND } D0 (\text{NOR } D0 D0)) = 0$$



- Constants help form introns but may be disrupted by crossover.
- However large subtrees which always output either 0 or 1 tend to be resilient to crossover

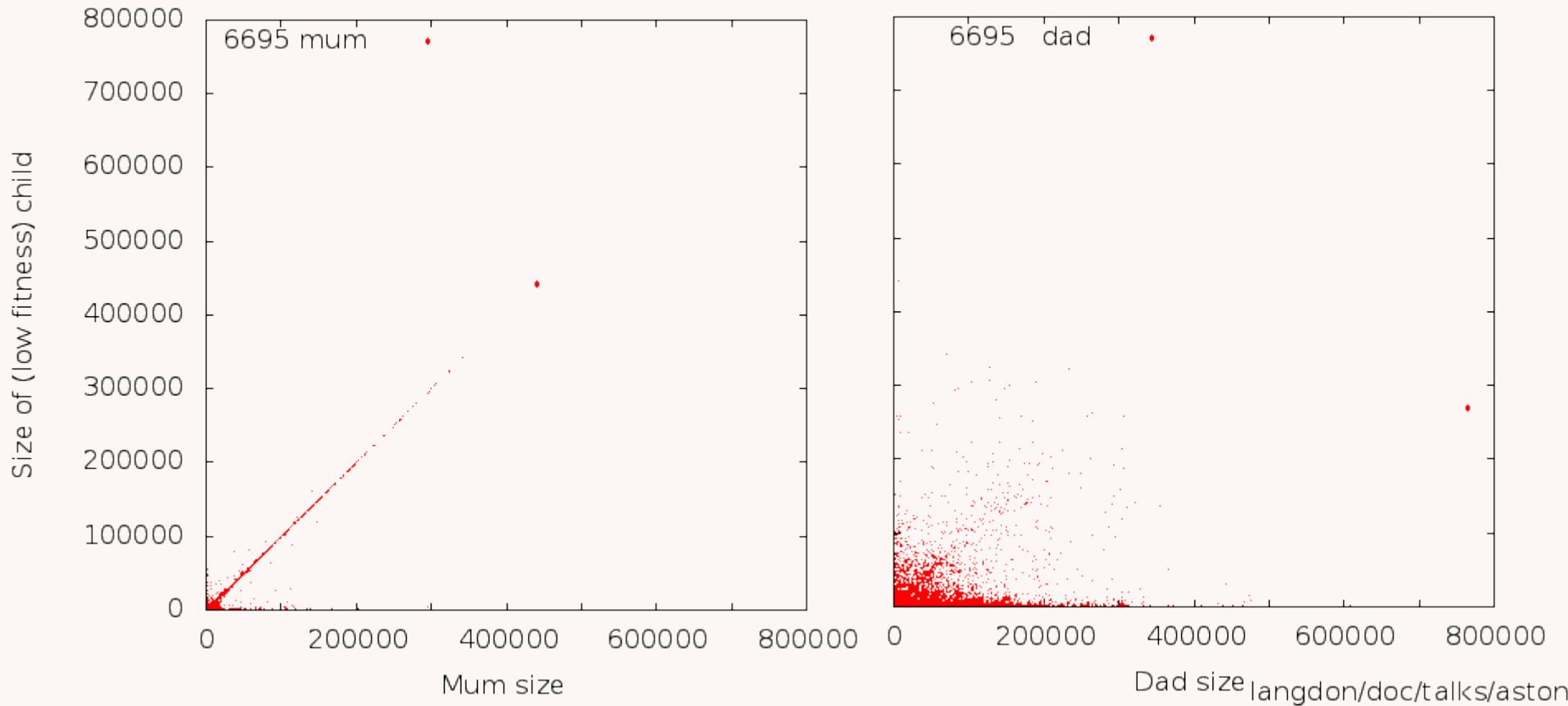
# Runts Drive Evolution



Don't plot ratio if less than 5 data

# Importance of Mothers

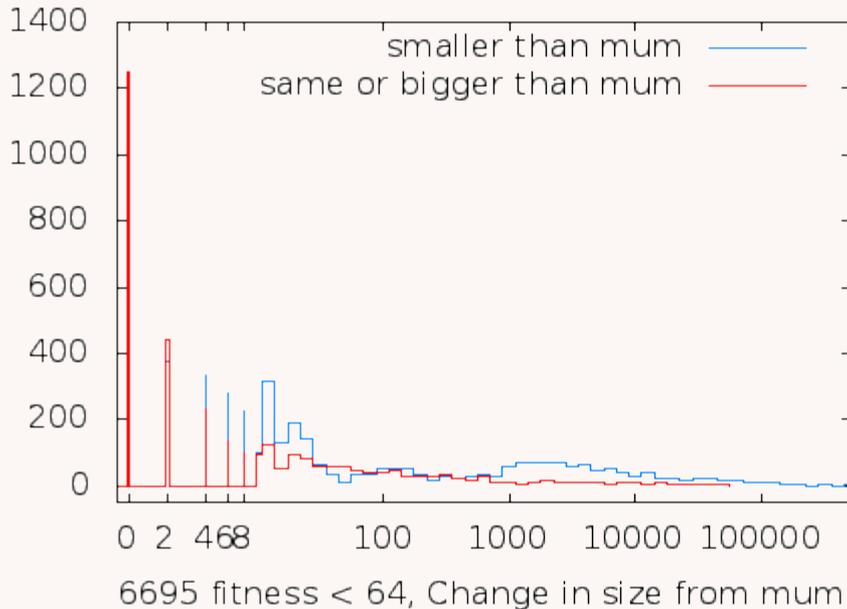
All children fitness <64 when both parents fit=64    All children fitness <64 when both parents fit=64



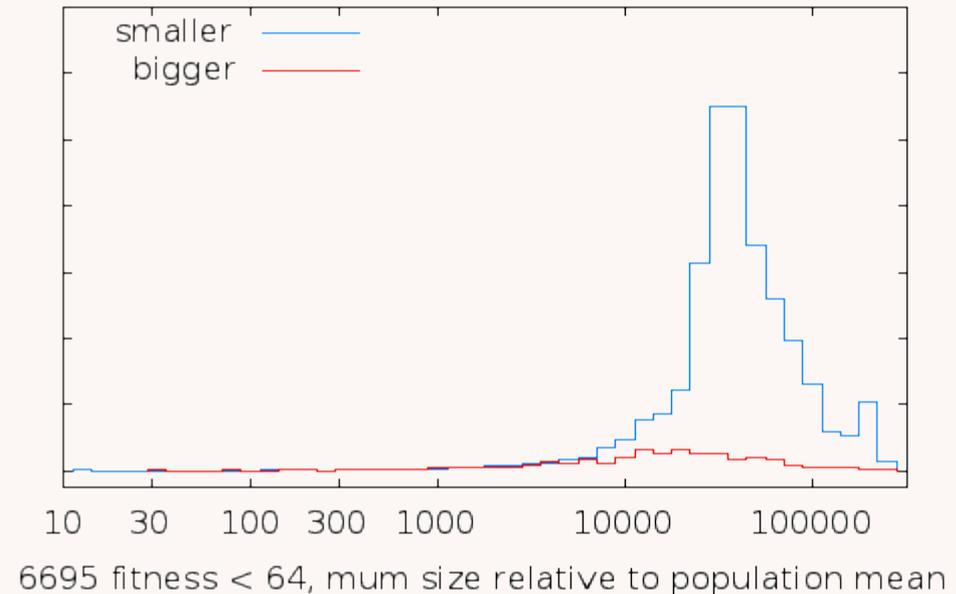
Size of poor fitness children closely related to parent who they inherit root from (mum).

# Importance of Mothers

All children fitness  $< 64$  when both parents fit = 64

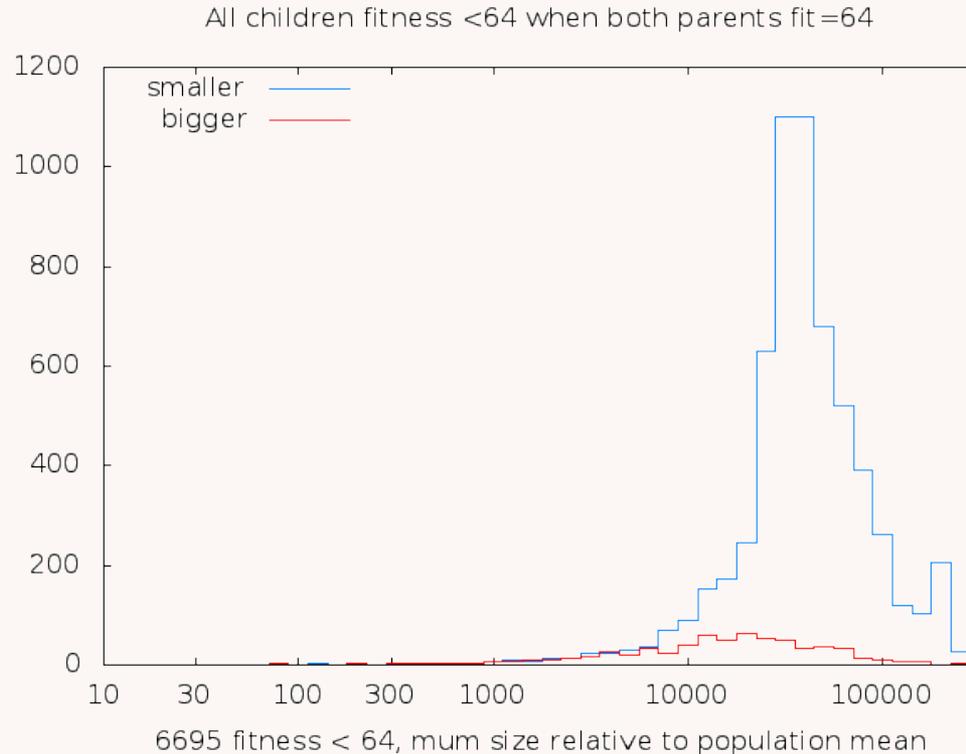


All children fitness  $< 64$  when both parents fit = 64



- Although many runts are smaller than their mum,
- many mothers of runts are smaller than average.
- Selection removes all low fitness children,
- Since these are smaller than average, the average size increases

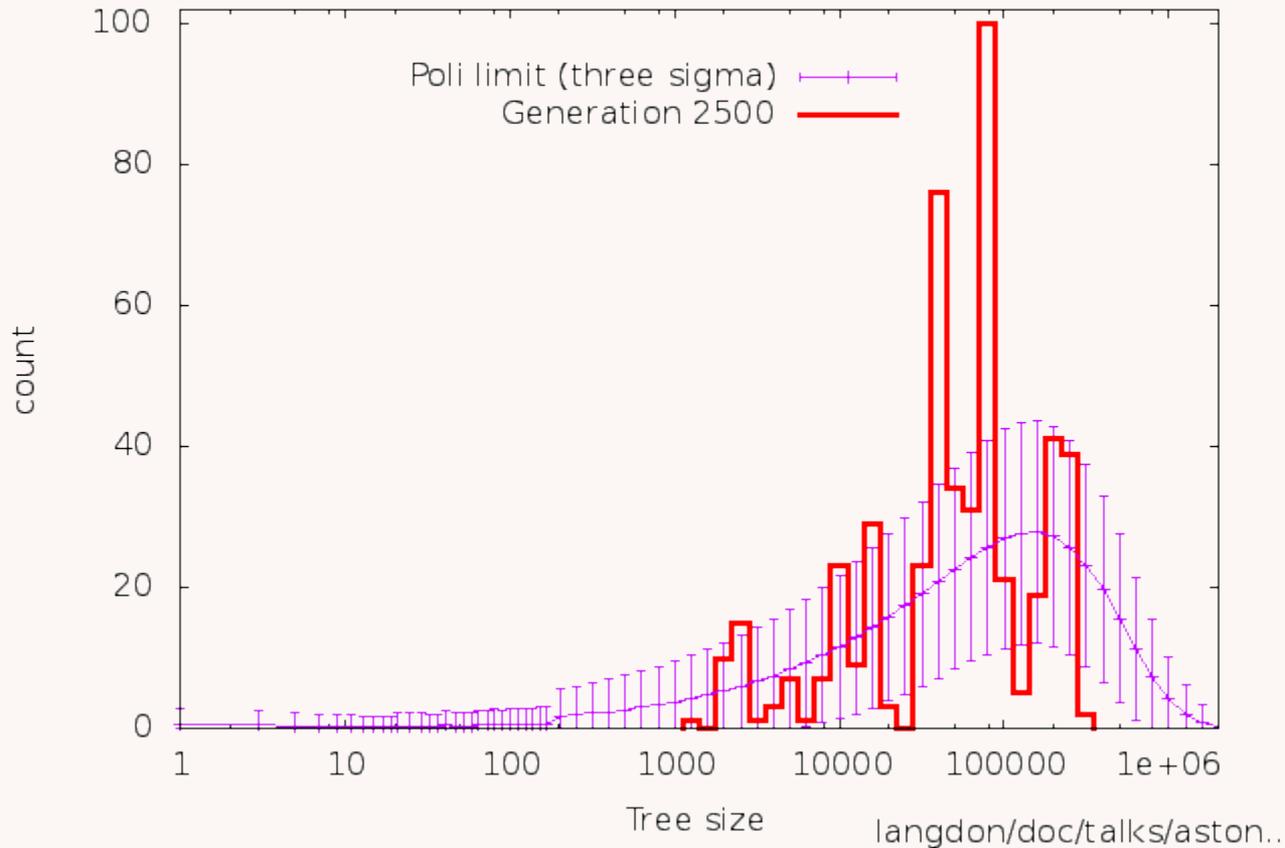
# A few runts drive size increase



- Many mothers of runts are smaller than average (blue)
- Selection removes all low fitness children (runts)
- Since these are smaller than average
- Although there is noise, on average size increases

# Testing Theory

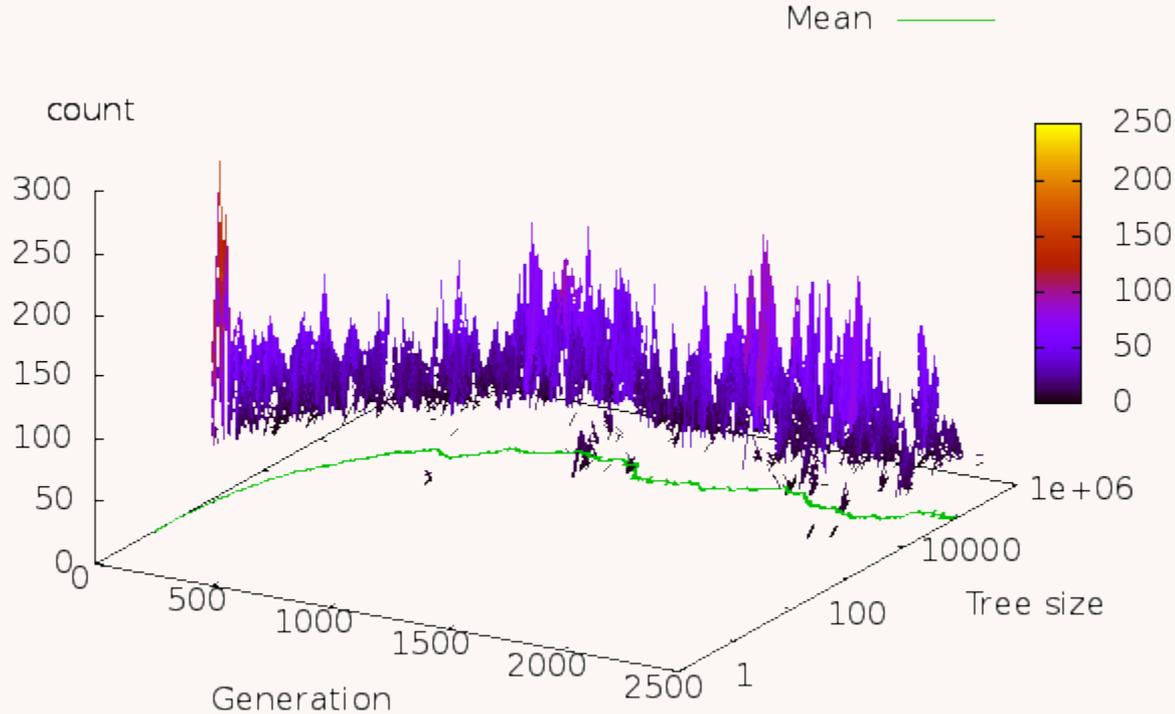
6-Mux 500 binary trees (run 100 at Gen 2500)



- Theory assumes crossover only (no selection). In earlier work distribution of sizes converged to limit rapidly.
- Selection caused by a few runts modifies size distribution

# Testing Theory

6-Mux 500 binary trees (run 100 up to Gen 2500)



- Same as testing theory plot but do every generation
- Colour only part of histogram  $\geq 3\sigma$
- Small tree and large tree tails ok (not coloured)

# The Genetic Programming Bibliography

<http://www.cs.bham.ac.uk/~wbl/biblio/>

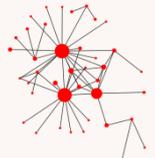
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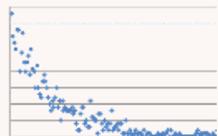
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